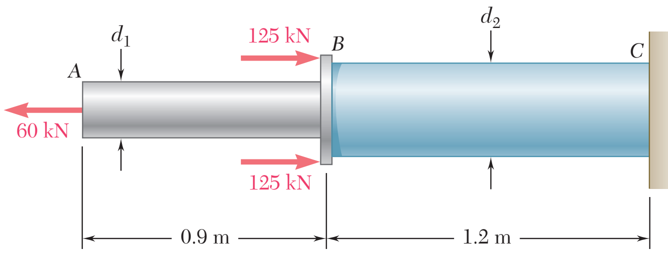
CHAPTER 1



PROBLEM 1.1

Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Knowing that $d_1 = 30$ mm and $d_2 = 50$ mm, find the average normal stress at the midsection of (*a*) rod *AB*, (*b*) rod *BC*.

SOLUTION

(*a*) Rod *AB*:

Force: $P = 60 \times 10^3$ N tension

Area: $A = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(30 \times 10^{-3})^2 = 706.86 \times 10^{-6}\ \text{m}^2$

Normal stress: $\sigma_{AB} = \frac{P}{A} = \frac{60 \times 10^3}{706.86 \times 10^{-6}} = 84.882 \times 10^6$ Pa

$\sigma_{AB} = 84.9$ MPa ◀

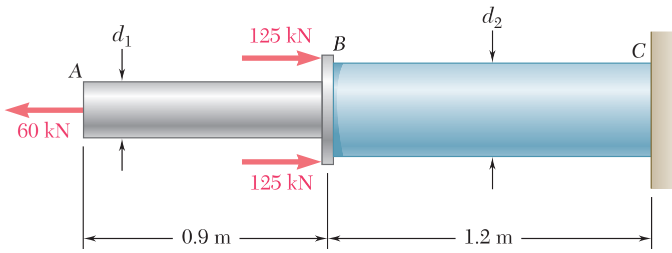
(*b*) Rod *BC*:

Force: $P = 60 \times 10^3 - (2)(125 \times 10^3) = -190 \times 10^3$ N

Area: $A = \frac{\pi}{4} d_2^2 = \frac{\pi}{4}(50 \times 10^{-3})^2 = 1.96350 \times 10^{-3}\ \text{m}^2$

Normal stress: $\sigma_{BC} = \frac{P}{A} = \frac{-190 \times 10^3}{1.96350 \times 10^{-3}} = -96.766 \times 10^6$ Pa

$\sigma_{BC} = -96.8$ MPa ◀



PROBLEM 1.2

Two solid cylindrical rods AB and BC are welded together at B and loaded as shown. Knowing that the average normal stress must not exceed 150 MPa in either rod, determine the smallest allowable values of the diameters d_1 and d_2.

SOLUTION

(*a*) Rod AB:

Force: $$P = 60 \times 10^3 \text{ N}$$

Stress: $$\sigma_{AB} = 150 \times 10^6 \text{ Pa}$$

Area: $$A = \frac{\pi}{4} d_1^2$$

$$\sigma_{AB} = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma_{AB}}$$

$$\frac{\pi}{4} d_1^2 = \frac{P}{\sigma_{AB}}$$

$$d_1^2 = \frac{4P}{\pi \sigma_{AB}} = \frac{(4)(60 \times 10^3)}{\pi (150 \times 10^6)} = 509.30 \times 10^{-6} \text{ m}^2$$

$$d_1 = 22.568 \times 10^{-3} \text{ m}$$

$d_1 = 22.6$ mm ◀

(*b*) Rod BC:

Force: $$P = 60 \times 10^3 - (2)(125 \times 10^3) = -190 \times 10^3 \text{ N}$$

Stress: $$\sigma_{BC} = -150 \times 10^6 \text{ Pa}$$

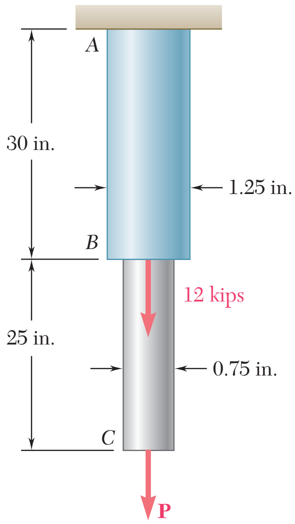
Area: $$A = \frac{\pi}{4} d_2^2$$

$$\sigma_{BC} = \frac{P}{A} = \frac{4P}{\pi d_2^2}$$

$$d_2^2 = \frac{4P}{\pi \sigma_{BC}} = \frac{(4)(-190 \times 10^3)}{\pi (-150 \times 10^6)} = 1.61277 \times 10^{-3} \text{ m}^2$$

$$d_2 = 40.159 \times 10^{-3} \text{ m}$$

$d_2 = 40.2$ mm ◀



PROBLEM 1.3

Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Knowing that $P = 10$ kips, find the average normal stress at the midsection of (*a*) rod *AB*, (*b*) rod *BC*.

SOLUTION

(*a*) Rod *AB*:

$$P = 12 + 10 = 22 \text{ kips}$$

$$A = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(1.25)^2 = 1.22718 \text{ in}^2$$

$$\sigma_{AB} = \frac{P}{A} = \frac{22}{1.22718} = 17.927 \text{ ksi}$$

$\sigma_{AB} = 17.93$ ksi ◀

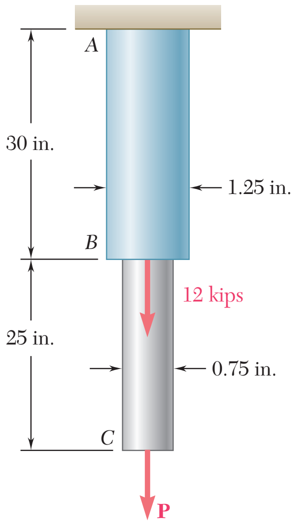
(*b*) Rod *BC*:

$$P = 10 \text{ kips}$$

$$A = \frac{\pi}{4} d_2^2 = \frac{\pi}{4}(0.75)^2 = 0.44179 \text{ in}^2$$

$$\sigma_{AB} = \frac{P}{A} = \frac{10}{0.44179} = 22.635 \text{ ksi}$$

$\sigma_{AB} = 22.6$ ksi ◀



PROBLEM 1.4

Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Determine the magnitude of the force **P** for which the tensile stresses in rods *AB* and *BC* are equal.

SOLUTION

(*a*) Rod *AB*:

$$P = P + 12 \text{ kips}$$

$$A = \frac{\pi d^2}{4} = \frac{\pi}{4}(1.25 \text{ in.})^2$$

$$A = 1.22718 \text{ in}^2$$

$$\sigma_{AB} = \frac{P + 12 \text{ kips}}{1.22718 \text{ in}^2}$$

(*b*) Rod *BC*:

$$P = P$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.75 \text{ in.})^2$$

$$A = 0.44179 \text{ in}^2$$

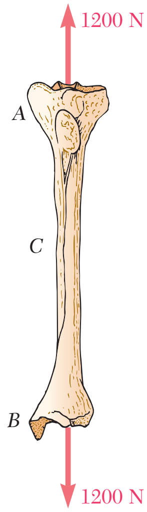
$$\sigma_{BC} = \frac{P}{0.44179 \text{ in}^2}$$

$$\sigma_{AB} = \sigma_{BC}$$

$$\frac{P + 12 \text{ kips}}{1.22718 \text{ in}^2} = \frac{P}{0.44179 \text{ in}^2}$$

$$5.3015 = 0.78539P$$

$P = 6.75$ kips ◀



PROBLEM 1.5

A strain gage located at C on the surface of bone AB indicates that the average normal stress in the bone is 3.80 MPa when the bone is subjected to two 1200-N forces as shown. Assuming the cross section of the bone at C to be annular and knowing that its outer diameter is 25 mm, determine the inner diameter of the bone's cross section at C.

SOLUTION

$$\sigma = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma}$$

Geometry: $A = \frac{\pi}{4}(d_1^2 - d_2^2)$

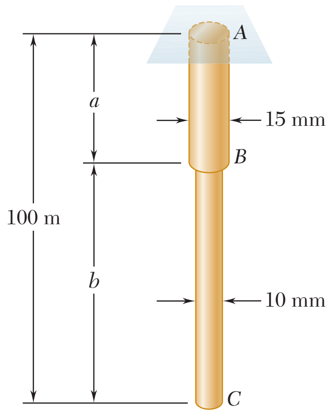
$$d_2^2 = d_1^2 - \frac{4A}{\pi} = d_1^2 - \frac{4P}{\pi\sigma}$$

$$d_2^2 = (25 \times 10^{-3})^2 - \frac{(4)(1200)}{\pi(3.80 \times 10^6)}$$

$$= 222.92 \times 10^{-6}\ \text{m}^2$$

$$d_2 = 14.93 \times 10^{-3}\ \text{m}$$

$d_2 = 14.93$ mm ◀



PROBLEM 1.6

Two brass rods *AB* and *BC*, each of uniform diameter, will be brazed together at *B* to form a nonuniform rod of total length 100 m, which will be suspended from a support at *A* as shown. Knowing that the density of brass is 8470 kg/m^3, determine (*a*) the length of rod *AB* for which the maximum normal stress in *ABC* is minimum, (*b*) the corresponding value of the maximum normal stress.

SOLUTION

Areas: $A_{AB} = \frac{\pi}{4}(15\text{ mm})^2 = 176.715\text{ mm}^2 = 176.715 \times 10^{-6}\text{m}^2$

$A_{BC} = \frac{\pi}{4}(10\text{ mm})^2 = 78.54\text{ mm}^2 = 78.54 \times 10^{-6}\text{m}^2$

From geometry, $b = 100 - a$

Weights: $W_{AB} = \rho g\, A_{AB}\ell_{AB} = (8470)(9.81)(176.715 \times 10^{-6})a = 14.683a$

$W_{BC} = \rho g\, A_{BC}\ell_{BC} = (8470)(9.81)(78.54 \times 10^{-6})(100 - a) = 652.59 - 6.526a$

Normal stresses:

At *A*, $P_A = W_{AB} + W_{BC} = 652.59 + 8.157a$ (1)

$$\sigma_A = \frac{P_A}{A_{AB}} = 3.6930 \times 10^6 + 46.160 \times 10^3 a$$

At *B*, $P_B = W_{BC} = 652.59 - 6.526a$ (2)

$$\sigma_B = \frac{P_B}{A_{BC}} = 8.3090 \times 10^6 - 83.090 \times 10^3 a$$

(*a*) Length of rod *AB*. The maximum stress in *ABC* is minimum when $\sigma_A = \sigma_B$ or

$$4.6160 \times 10^6 - 129.25 \times 10^3 a = 0$$

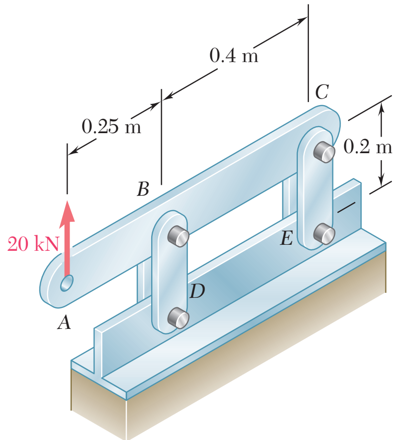
$a = 35.71\text{ m}$ $\ell_{AB} = a = 35.7\text{ m}$ ◀

(*b*) Maximum normal stress.

$$\sigma_A = 3.6930 \times 10^6 + (46.160 \times 10^3)(35.71)$$

$$\sigma_B = 8.3090 \times 10^6 - (83.090 \times 10^3)(35.71)$$

$\sigma_A = \sigma_B = 5.34 \times 10^6\text{ Pa}$ $\sigma = 5.34\text{ MPa}$ ◀

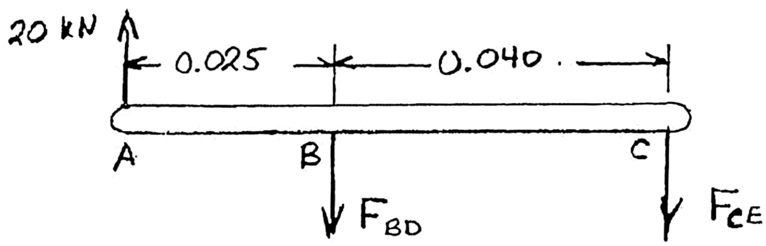


PROBLEM 1.7

Each of the four vertical links has an 8×36-mm uniform rectangular cross section and each of the four pins has a 16-mm diameter. Determine the maximum value of the average normal stress in the links connecting (*a*) points *B* and *D*, (*b*) points *C* and *E*.

SOLUTION

Use bar *ABC* as a free body.



$\Sigma M_C = 0: \quad (0.040) F_{BD} - (0.025 + 0.040)(20 \times 10^3) = 0$

$F_{BD} = 32.5 \times 10^3 \text{ N}$ Link *BD* is in tension.

$\Sigma M_B = 0: \quad -(0.040) F_{CE} - (0.025)(20 \times 10^3) = 0$

$F_{CE} = -12.5 \times 10^3 \text{ N}$ Link *CE* is in compression.

Net area of one link for tension $= (0.008)(0.036 - 0.016) = 160 \times 10^{-6} \text{ m}^2$

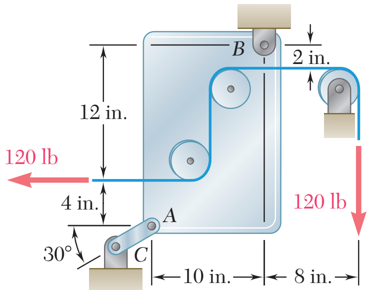
For two parallel links, $A_{\text{net}} = 320 \times 10^{-6} \text{ m}^2$

(*a*) $\sigma_{BD} = \dfrac{F_{BD}}{A_{\text{net}}} = \dfrac{32.5 \times 10^3}{320 \times 10^{-6}} = 101.563 \times 10^6$ $\qquad \sigma_{BD} = 101.6 \text{ MPa}$ ◀

Area for one link in compression $= (0.008)(0.036) = 288 \times 10^{-6} \text{ m}^2$

For two parallel links, $A = 576 \times 10^{-6} \text{ m}^2$

(*b*) $\sigma_{CE} = \dfrac{F_{CE}}{A} = \dfrac{-12.5 \times 10^3}{576 \times 10^{-6}} = -21.701 \times 10^{-6}$ $\qquad \sigma_{CE} = -21.7 \text{ MPa}$ ◀

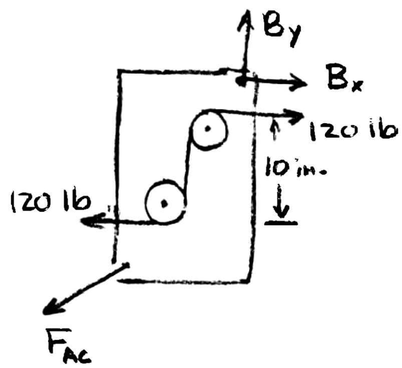


PROBLEM 1.8

Link AC has a uniform rectangular cross section $\frac{1}{8}$ in. thick and 1 in. wide. Determine the normal stress in the central portion of the link.

SOLUTION

Use the plate together with two pulleys as a free body. Note that the cable tension causes at 1200 lb-in. clockwise couple to act on the body.

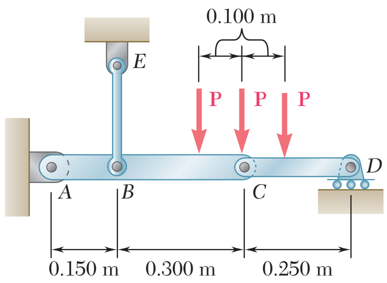


$$+\circlearrowleft \Sigma M_B = 0: \quad -(12+4)(F_{AC}\cos 30°) + (10)(F_{AC}\sin 30°) - 1200 \text{ lb} = 0$$

$$F_{AC} = -\frac{1200 \text{ lb}}{16\cos 30° - 10\sin 30°} = -135.500 \text{ lb}$$

Area of link AC: $\quad A = 1 \text{ in.} \times \frac{1}{8} \text{ in.} = 0.125 \text{ in}^2$

Stress in link AC: $\quad \sigma_{AC} = \frac{F_{AC}}{A} = -\frac{135.50}{0.125} = 1084 \text{ psi} = 1.084 \text{ ksi}$ ◀

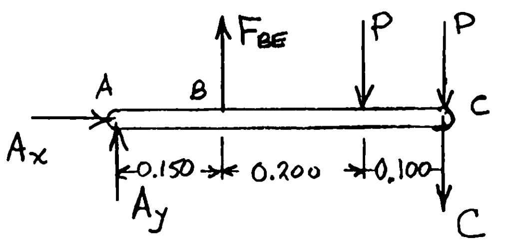


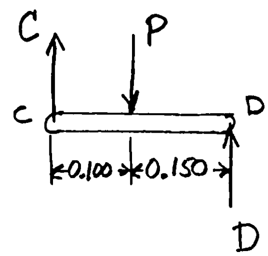
PROBLEM 1.9

Three forces, each of magnitude $P = 4$ kN, are applied to the mechanism shown. Determine the cross-sectional area of the uniform portion of rod BE for which the normal stress in that portion is +100 MPa.

SOLUTION

Draw free body diagrams of AC and CD.





Free Body CD: $\quad +\circlearrowleft \Sigma M_D = 0: \quad 0.150P - 0.250C = 0$

$$C = 0.6P$$

Free Body AC: $\quad +\circlearrowleft M_A = 0: \quad 0.150F_{BE} - 0.350P - 0.450P - 0.450C = 0$

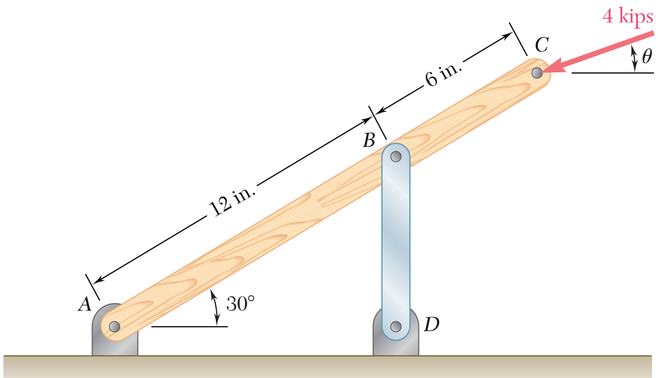
$$F_{BE} = \frac{1.07}{0.150}P = 7.1333\,P = (7.133)(4\text{ kN}) = 28.533\text{ kN}$$

Required area of BE:

$$\sigma_{BE} = \frac{F_{BE}}{A_{BE}}$$

$$A_{BE} = \frac{F_{BE}}{\sigma_{BE}} = \frac{28.533 \times 10^3}{100 \times 10^6} = 285.33 \times 10^{-6}\text{m}^2$$

$$A_{BE} = 285\text{ mm}^2 \blacktriangleleft$$

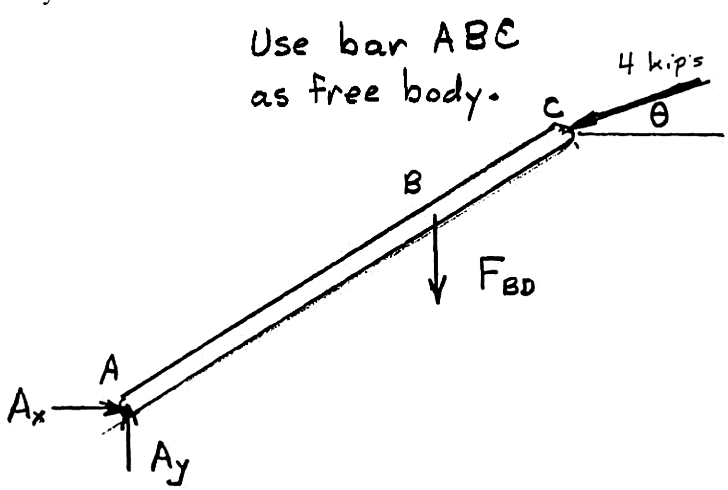


PROBLEM 1.10

Link BD consists of a single bar 1 in. wide and $\frac{1}{2}$ in. thick. Knowing that each pin has a $\frac{3}{8}$-in. diameter, determine the maximum value of the average normal stress in link BD if (*a*) $\theta = 0$, (*b*) $\theta = 90°$.

SOLUTION

Use bar *ABC* as a free body.



(*a*) $\underline{\theta = 0.}$

$$+\circlearrowleft \Sigma M_A = 0: \quad (18 \sin 30°)(4) - (12 \cos 30°)F_{BD} = 0$$

$$F_{BD} = 3.4641 \text{ kips} \quad \text{(tension)}$$

Area for tension loading: $$A = (b - d)t = \left(1 - \frac{3}{8}\right)\left(\frac{1}{2}\right) = 0.31250 \text{ in}^2$$

Stress: $$\sigma = \frac{F_{BD}}{A} = \frac{3.4641 \text{ kips}}{0.31250 \text{ in}^2}$$ $\sigma = 11.09 \text{ ksi}$ ◀

(*b*) $\underline{\theta = 90°.}$

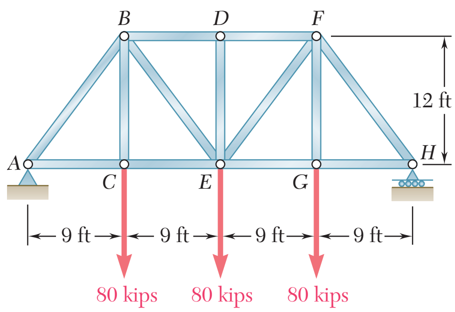
$$+\circlearrowleft \Sigma M_A = 0: \quad -(18 \cos 30°)(4) - (12 \cos 30°)F_{BD} = 0$$

$$F_{BD} = -6 \text{ kips} \quad \text{i.e. compression.}$$

Area for compression loading: $$A = bt = (1)\left(\frac{1}{2}\right) = 0.5 \text{ in}^2$$

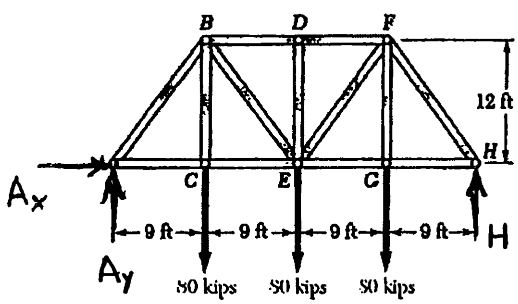
Stress: $$\sigma = \frac{F_{BD}}{A} = \frac{-6 \text{ kips}}{0.5 \text{ in}^2}$$ $\sigma = 12.00 \text{ ksi}$ ◀

PROBLEM 1.11



For the Pratt bridge truss and loading shown, determine the average normal stress in member *BE*, knowing that the cross-sectional area of that member is 5.87 in^2.

SOLUTION



Use entire truss as free body.

$$+\circlearrowleft \Sigma M_H = 0: \quad (9)(80) + (18)(80) + (27)(80) - 36A_y = 0$$

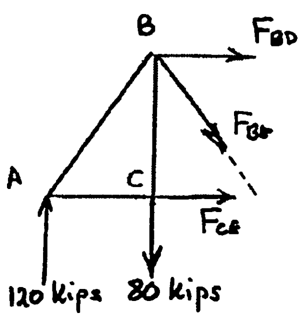
$$A_y = 120 \text{ kips}$$

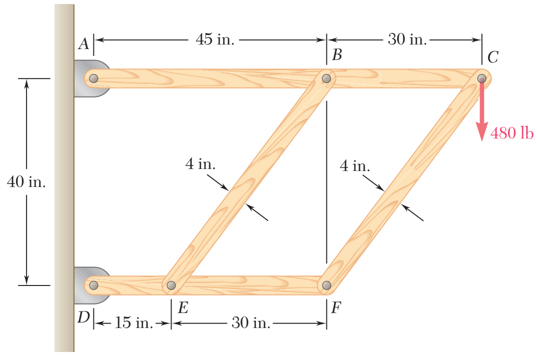
Use portion of truss to the left of a section cutting members *BD*, *BE*, and *CE*.

$$+\uparrow \Sigma F_y = 0: \quad 120 - 80 - \frac{12}{15}F_{BE} = 0 \qquad \therefore\ F_{BE} = 50 \text{ kips}$$

$$\sigma_{BE} = \frac{F_{BE}}{A} = \frac{50 \text{ kips}}{5.87 \text{ in}^2}$$

$$\sigma_{BE} = 8.52 \text{ ksi} \blacktriangleleft$$

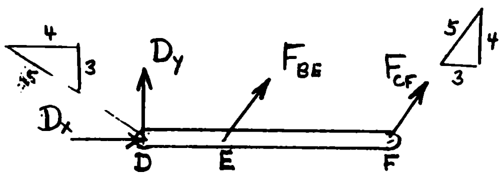




PROBLEM 1.12

The frame shown consists of *four* wooden members, *ABC, DEF, BE,* and *CF*. Knowing that each member has a 2 × 4-in. rectangular cross section and that each pin has a $\frac{1}{2}$-in. diameter, determine the maximum value of the average normal stress (*a*) in member *BE*, (*b*) in member *CF*.

SOLUTION



Add support reactions to figure as shown.

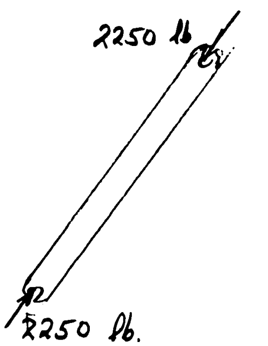
Using entire frame as free body,

$$\Sigma M_A = 0: \quad 40D_x - (45+30)(480) = 0$$

$$D_x = 900 \text{ lb}$$

Use member DEF as free body.

Reaction at D must be parallel to F_{BE} and F_{CF}.



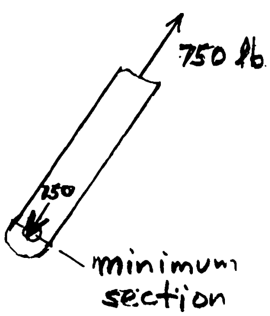
$$D_y = \frac{4}{3}D_x = 1200 \text{ lb}$$

$$\Sigma M_F = 0: \quad -(30)\left(\frac{4}{5}F_{BE}\right) - (30+15)D_Y = 0$$

$$F_{BE} = -2250 \text{ lb}$$

$$\Sigma M_E = 0: \quad (30)\left(\frac{4}{5}F_{CE}\right) - (15)D_Y = 0$$

$$F_{CE} = 750 \text{ lb}$$



Stress in compression member *BE*:

Area: $A = 2 \text{ in.} \times 4 \text{ in.} = 8 \text{ in}^2$

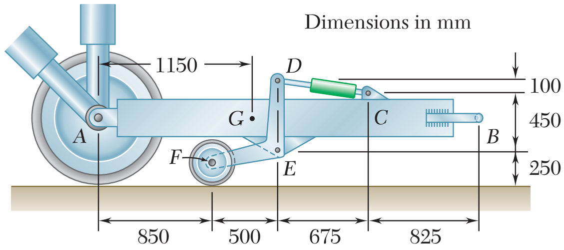
(*a*) $\sigma_{BE} = \dfrac{F_{BE}}{A} = \dfrac{-2250}{8}$ $\qquad \sigma_{BE} = -281 \text{ psi}$ ◀

Minimum section area occurs at pin.

$$A_{\min} = (2)(4.0 - 0.5) = 7.0 \text{ in}^2$$

Stress in tension member *CF*:

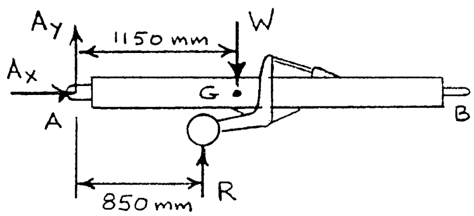
(*b*) $\sigma_{CF} = \dfrac{F_{CF}}{A_{\min}} = \dfrac{750}{7.0}$ $\qquad \sigma_{CF} = 107.1 \text{ psi}$ ◀



PROBLEM 1.13

An aircraft tow bar is positioned by means of a single hydraulic cylinder connected by a 25-mm-diameter steel rod to two identical arm-and-wheel units *DEF*. The mass of the entire tow bar is 200 kg, and its center of gravity is located at *G*. For the position shown, determine the normal stress in the rod.

SOLUTION

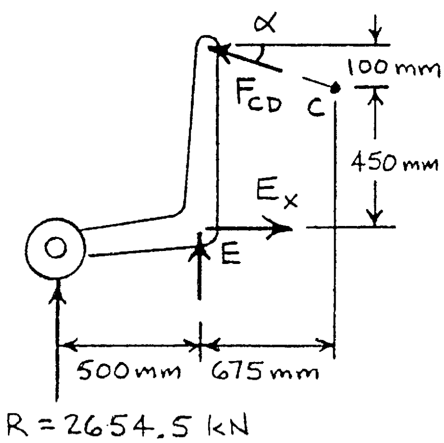


FREE BODY – ENTIRE TOW BAR:

$$W = (200 \text{ kg})(9.81 \text{ m/s}^2) = 1962.00 \text{ N}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 850R - 1150(1962.00 \text{ N}) = 0$$

$$R = 2654.5 \text{ N}$$



FREE BODY – BOTH ARM & WHEEL UNITS:

$$\tan\alpha = \frac{100}{675} \qquad \alpha = 8.4270°$$

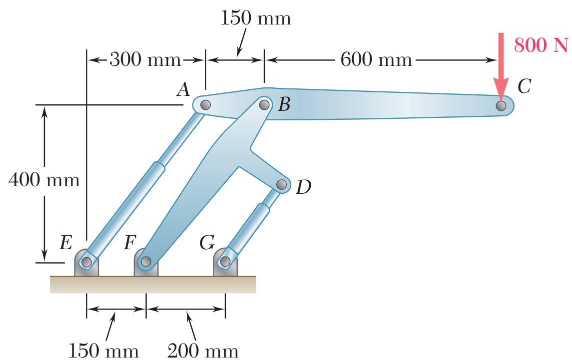
$$+\circlearrowleft \Sigma M_E = 0: \quad (F_{CD}\cos\alpha)(550) - R(500) = 0$$

$$F_{CD} = \frac{500}{550 \cos 8.4270°}(2654.5 \text{ N})$$

$$= 2439.5 \text{ N} \quad \text{(comp.)}$$

$$\sigma_{CD} = -\frac{F_{CD}}{A_{CD}} = -\frac{2439.5 \text{ N}}{\pi(0.0125 \text{ m})^2}$$

$$= -4.9697 \times 10^6 \text{Pa} \qquad \sigma_{CD} = -4.97 \text{ MPa} \blacktriangleleft$$

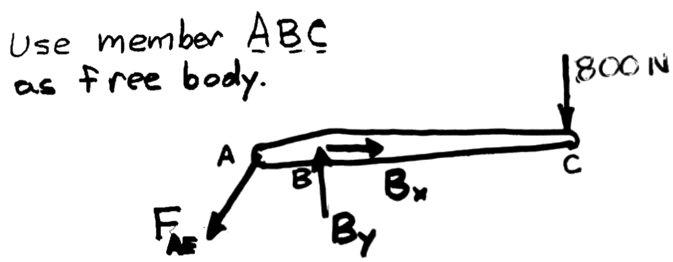


PROBLEM 1.14

Two hydraulic cylinders are used to control the position of the robotic arm *ABC*. Knowing that the control rods attached at *A* and *D* each have a 20-mm diameter and happen to be parallel in the position shown, determine the average normal stress in (*a*) member *AE*, (*b*) member *DG*.

SOLUTION

Use member *ABC* as free body.



$$+\circlearrowleft \Sigma M_B = 0: \quad (0.150)\frac{4}{5}F_{AE} - (0.600)(800) = 0$$

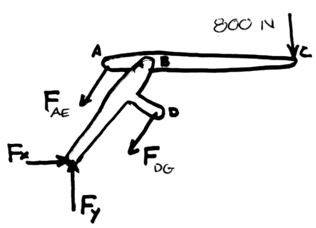
$$F_{AE} = 4 \times 10^3 \text{ N}$$

Area of rod in member *AE* is $\quad A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(20 \times 10^{-3})^2 = 314.16 \times 10^{-6} \text{m}^2$

Stress in rod *AE*: $\quad \sigma_{AE} = \frac{F_{AE}}{A} = \frac{4 \times 10^3}{314.16 \times 10^{-6}} = 12.7324 \times 10^6 \text{ Pa}$

(*a*) $\quad \sigma_{AE} = 12.73 \text{ MPa}$ ◀

Use combined members *ABC* and *BFD* as free body.



$$+\circlearrowleft \Sigma M_F = 0: \quad (0.150)\left(\frac{4}{5}F_{AE}\right) - (0.200)\left(\frac{4}{5}F_{DG}\right) - (1.050 - 0.350)(800) = 0$$

$$F_{DG} = -1500 \text{ N}$$

Area of rod *DG*: $\quad A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(20 \times 10^{-3})^2 = 314.16 \times 10^{-6} \text{ m}^2$

Stress in rod *DG*: $\quad \sigma_{DG} = \frac{F_{DG}}{A} = \frac{-1500}{3.1416 \times 10^{-6}} = -4.7746 \times 10^6 \text{ Pa}$

(*b*) $\quad \sigma_{DG} = -4.77 \text{ MPa}$ ◀

PROBLEM 1.15

Determine the diameter of the largest circular hole that can be punched into a sheet of polystyrene 6 mm thick, knowing that the force exerted by the punch is 45 kN and that a 55-MPa average shearing stress is required to cause the material to fail.

SOLUTION

For cylindrical failure surface: $A = \pi d t$

Shearing stress: $\tau = \dfrac{P}{A}$ or $A = \dfrac{P}{\tau}$

Therefore, $\dfrac{P}{\tau} = \pi d t$

Finally,

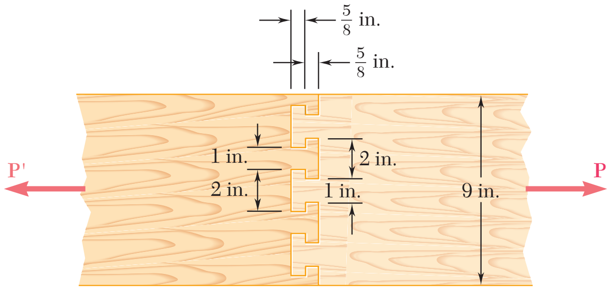
$$d = \frac{P}{\pi t \tau}$$

$$= \frac{45 \times 10^3 \text{ N}}{\pi(0.006 \text{ m})(55 \times 10^6 \text{ Pa})}$$

$$= 43.406 \times 10^{-3} \text{ m}$$

$d = 43.4$ mm ◀

PROBLEM 1.16



Two wooden planks, each $\frac{1}{2}$ in. thick and 9 in. wide, are joined by the dry mortise joint shown. Knowing that the wood used shears off along its grain when the average shearing stress reaches 1.20 ksi, determine the magnitude P of the axial load that will cause the joint to fail.

SOLUTION

Six areas must be sheared off when the joint fails. Each of these areas has dimensions $\frac{5}{8}$ in. $\times$ $\frac{1}{2}$ in., its area being

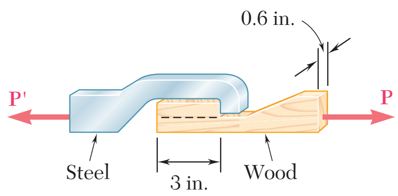
$$A = \frac{5}{8} \times \frac{1}{2} = \frac{5}{16} \text{ in}^2 = 0.3125 \text{ in}^2$$

At failure, the force carried by each area is

$$F = \tau A = (1.20 \text{ ksi})(0.3125 \text{ in}^2) = 0.375 \text{ kips}$$

Since there are six failure areas,

$$P = 6F = (6)(0.375) \qquad P = 2.25 \text{ kips} \blacktriangleleft$$



PROBLEM 1.17

When the force **P** reached 1600 lb, the wooden specimen shown failed in shear along the surface indicated by the dashed line. Determine the average shearing stress along that surface at the time of failure.

SOLUTION

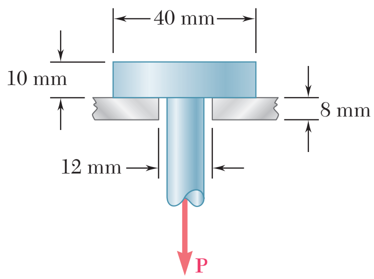
Area being sheared: $A = 3 \text{ in.} \times 0.6 \text{ in.} = 1.8 \text{ in}^2$

Force: $P = 1600 \text{ lb}$

Shearing stress: $\tau = \dfrac{P}{A} - \dfrac{1600 \text{ lb}}{1.8 \text{ in}^2} = 8.8889 \times 10^2 \text{ psi}$

$\tau = 889 \text{ psi}$ ◀

PROBLEM 1.18



A load **P** is applied to a steel rod supported as shown by an aluminum plate into which a 12-mm-diameter hole has been drilled. Knowing that the shearing stress must not exceed 180 MPa in the steel rod and 70 MPa in the aluminum plate, determine the largest load **P** that can be applied to the rod.

SOLUTION

For steel:

$$A_1 = \pi d t = \pi(0.012 \text{ m})(0.010 \text{ m})$$

$$= 376.99 \times 10^{-6} \text{ m}^2$$

$$\tau_1 = \frac{P}{A} \therefore P = A_1 \tau_1 = (376.99 \times 10^{-6} \text{ m}^2)(180 \times 10^6 \text{ Pa})$$

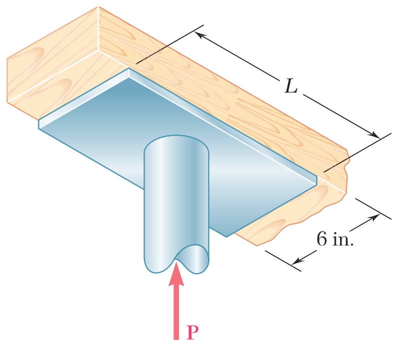
$$= 67.858 \times 10^3 \text{ N}$$

For aluminum:

$$A_2 = \pi d t = \pi(0.040 \text{ m})(0.008 \text{ m}) = 1.00531 \times 10^{-3} \text{ m}^2$$

$$\tau_2 = \frac{P}{A_2} \therefore P = A_2 \tau_2 = (1.00531 \times 10^{-3} \text{ m}^2)(70 \times 10^6 \text{ Pa}) = 70.372 \times 10^3 \text{ N}$$

Limiting value of P is the smaller value, so $P = 67.9 \text{ kN}$ ◀



PROBLEM 1.19

The axial force in the column supporting the timber beam shown is $P = 20$ kips. Determine the smallest allowable length L of the bearing plate if the bearing stress in the timber is not to exceed 400 psi.

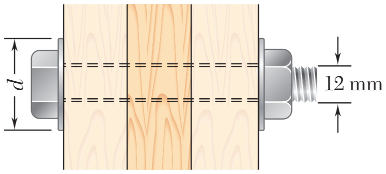
SOLUTION

Bearing area: $A_b = Lw$

$$\sigma_b = \frac{P}{A_b} = \frac{P}{Lw}$$

$$L = \frac{P}{\sigma_b w} = \frac{20 \times 10^3 \text{ lb}}{(400 \text{ psi})(6 \text{ in.})} = 8.33 \text{ in.}$$

$L = 8.33$ in. ◀



PROBLEM 1.20

Three wooden planks are fastened together by a series of bolts to form a column. The diameter of each bolt is 12 mm and the inner diameter of each washer is 16 mm, which is slightly larger than the diameter of the holes in the planks. Determine the smallest allowable outer diameter *d* of the washers, knowing that the average normal stress in the bolts is 36 MPa and that the bearing stress between the washers and the planks must not exceed 8.5 MPa.

SOLUTION

Bolt: $$A_{\text{Bolt}} = \frac{\pi d^2}{4} = \frac{\pi(0.012\text{ m})^2}{4} = 1.13097 \times 10^{-4}\text{ m}^2$$

Tensile force in bolt: $$\sigma = \frac{P}{A} \Rightarrow P = \sigma A$$

$$= (36 \times 10^6\text{ Pa})(1.13097 \times 10^{-4}\text{ m}^2)$$

$$= 4.0715 \times 10^3\text{ N}$$

Bearing area for washer: $$A_w = \frac{\pi}{4}\left(d_o^2 - d_i^2\right)$$

and $$A_w = \frac{P}{\sigma_{BRG}}$$

Therefore, equating the two expressions for A_w gives

$$\frac{\pi}{4}\left(d_o^2 - d_i^2\right) = \frac{P}{\sigma_{BRG}}$$

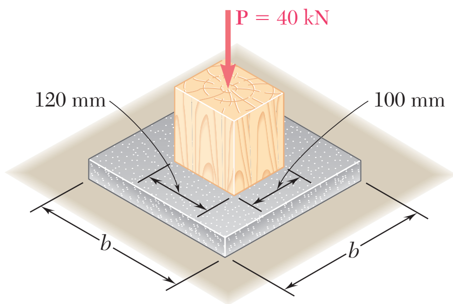
$$d_o^2 = \frac{4P}{\pi\sigma_{BRG}} + d_i^2$$

$$d_o^2 = \frac{4}{\pi}\frac{(4.0715 \times 10^3\text{ N})}{(8.5 \times 10^6\text{ Pa})} + (0.016\text{ m})^2$$

$$d_o^2 = 8.6588 \times 10^{-4}\text{ m}^2$$

$$d_o = 29.426 \times 10^{-3}\text{ m}$$

$d_o = 29.4$ mm ◀



PROBLEM 1.21

A 40-kN axial load is applied to a short wooden post that is supported by a concrete footing resting on undisturbed soil. Determine (*a*) the maximum bearing stress on the concrete footing, (*b*) the size of the footing for which the average bearing stress in the soil is 145 kPa.

SOLUTION

(*a*) Bearing stress on concrete footing.

$$P = 40 \text{ kN} = 40 \times 10^3 \text{ N}$$

$$A = (100)(120) = 12 \times 10^3 \text{ mm}^2 = 12 \times 10^{-3} \text{ m}^2$$

$$\sigma = \frac{P}{A} = \frac{40 \times 10^3}{12 \times 10^{-3}} = 3.3333 \times 10^6 \text{ Pa}$$

3.33 MPa ◀

(*b*) Footing area. $P = 40 \times 10^3$ N $\quad \sigma = 145 \text{ kPa} = 45 \times 10^3$ Pa

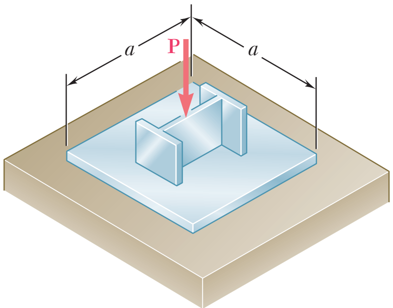
$$\sigma = \frac{P}{A} \quad A = \frac{P}{\sigma} = \frac{40 \times 10^3}{145 \times 10^3} = 0.27586 \text{ m}^2$$

Since the area is square, $A = b^2$

$$b = \sqrt{A} = \sqrt{0.27586} = 0.525 \text{ m}$$

$b = 525$ mm ◀

PROBLEM 1.22



An axial load **P** is supported by a short W8 × 40 column of cross-sectional area $A = 11.7 \text{ in}^2$ and is distributed to a concrete foundation by a square plate as shown. Knowing that the average normal stress in the column must not exceed 30 ksi and that the bearing stress on the concrete foundation must not exceed 3.0 ksi, determine the side a of the plate that will provide the most economical and safe design.

SOLUTION

For the column, $\sigma = \dfrac{P}{A}$ or

$$P = \sigma A = (30)(11.7) = 351 \text{ kips}$$

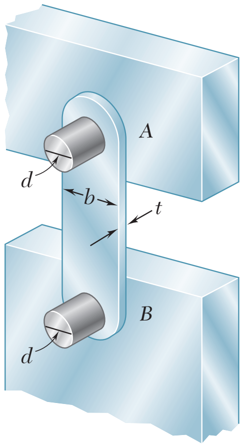
For the $a \times a$ plate, $\sigma = 3.0$ ksi

$$A = \frac{P}{\sigma} = \frac{351}{3.0} = 117 \text{ in}^2$$

Since the plate is square, $A = a^2$

$$a = \sqrt{A} = \sqrt{117}$$

$a = 10.82$ in. ◀



PROBLEM 1.23

Link AB, of width $b = 2$ in. and thickness $t = \frac{1}{4}$ in., is used to support the end of a horizontal beam. Knowing that the average normal stress in the link is −20 ksi and that the average shearing stress in each of the two pins is 12 ksi, determine (*a*) the diameter *d* of the pins, (*b*) the average bearing stress in the link.

SOLUTION

Rod *AB* is in compression.

$$A = bt \quad \text{where} \quad b = 2 \text{ in.} \quad \text{and} \quad t = \frac{1}{4} \text{ in.}$$

$$P = -\sigma A = -(-20)(2)\left(\frac{1}{4}\right) = 10 \text{ kips}$$

Pin: $\tau_P = \dfrac{P}{A_P}$

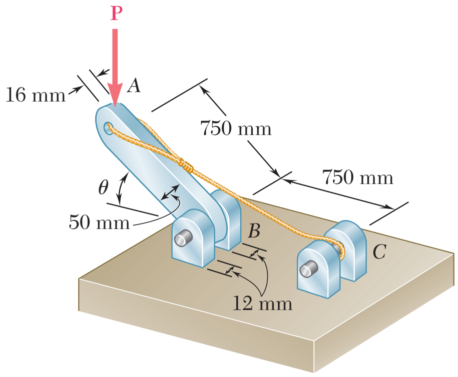
and $A_P = \dfrac{\pi}{4} d^2$

(*a*) $d = \sqrt{\dfrac{4A_P}{\pi}} = \sqrt{\dfrac{4P}{\pi\tau_P}} = \sqrt{\dfrac{(4)(10)}{\pi(12)}} = 1.03006 \text{ in.}$

$d = 1.030$ in. ◀

(*b*) $\sigma_b = \dfrac{P}{dt} = \dfrac{10}{(1.03006)(0.25)} = 38.833 \text{ ksi}$

$\sigma_b = 38.8$ ksi ◀

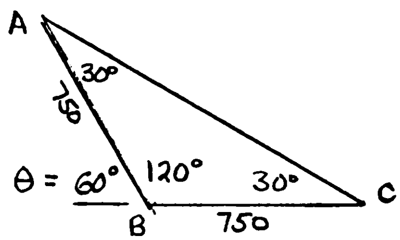


PROBLEM 1.24

Determine the largest load **P** which may be applied at A when $\theta = 60°$, knowing that the average shearing stress in the 10-mm-diameter pin at B must not exceed 120 MPa and that the average bearing stress in member AB and in the bracket at B must not exceed 90 MPa.

SOLUTION

Geometry: Triangle ABC is an isoseles triangle with angles shown here.



Use joint A as a free body.

Law of sines applied to force triangle:

$$\frac{P}{\sin 30°} = \frac{F_{AB}}{\sin 120°} = \frac{F_{AC}}{\sin 30°}$$

$$P = \frac{F_{AB} \sin 30°}{\sin 120°} = 0.57735 F_{AB}$$

$$P = \frac{F_{AC} \sin 30°}{\sin 30°} = F_{AC}$$

PROBLEM 1.24 *(Continued)*

If shearing stress in pin at B is critical,

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.010)^2 = 78.54 \times 10^{-6}\ \text{m}^2$$

$$F_{AB} = 2A\tau = (2)(78.54 \times 10^{-6})(120 \times 10^6) = 18.850 \times 10^3\ \text{N}$$

If bearing stress in member AB at bracket at A is critical,

$$A_b = td = (0.016)(0.010) = 160 \times 10^{-6}\ \text{m}^2$$

$$F_{AB} = A_b\sigma_b = (160 \times 10^{-6})(90 \times 10^6) = 14.40 \times 10^3\ \text{N}$$

If bearing stress in the bracket at B is critical,

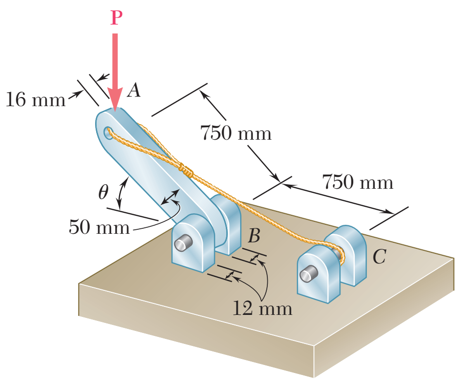
$$A_b = 2td = (2)(0.012)(0.010) = 240 \times 10^{-6}\ \text{m}^2$$

$$F_{AB} = A_b\sigma_b = (240 \times 10^{-6})(90 \times 10^6) = 21.6 \times 10^3\ \text{N}$$

Allowable F_{AB} is the smallest, i.e., 14.40×10^3 N

Then from statics, $P_{\text{allow}} = (0.57735)(14.40 \times 10^3)$

$= 8.31 \times 10^3\ \text{N}$ 8.31 kN ◀

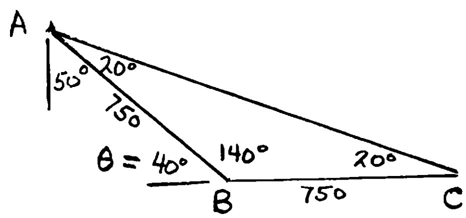


PROBLEM 1.25

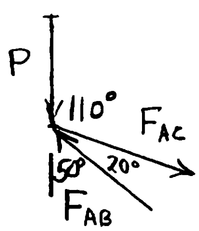
Knowing that $\theta = 40°$ and $P = 9$ kN, determine (*a*) the smallest allowable diameter of the pin at *B* if the average shearing stress in the pin is not to exceed 120 MPa, (*b*) the corresponding average bearing stress in member *AB* at *B*, (*c*) the corresponding average bearing stress in each of the support brackets at *B*.

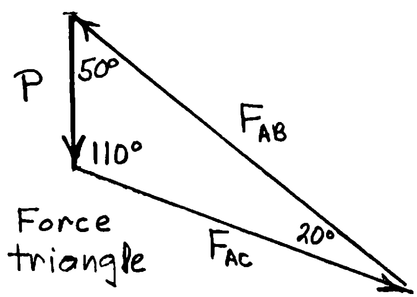
SOLUTION

Geometry: Triangle *ABC* is an isosceles triangle with angles shown here.



Use joint A as a free body.





Law of sines applied to force triangle:

$$\frac{P}{\sin 20°} = \frac{F_{AB}}{\sin 110°} = \frac{F_{AC}}{\sin 50°}$$

$$F_{AB} = \frac{P \sin 110°}{\sin 20°}$$

$$= \frac{(9)\sin 110°}{\sin 20°} = 24.727 \text{ kN}$$

PROBLEM 1.25 *(Continued)*

(*a*) Allowable pin diameter.

$$\tau = \frac{F_{AB}}{2A_P} = \frac{F_{AB}}{2\frac{\pi}{4}d^2} = \frac{2F_{AB}}{\pi d^2} \quad \text{where } F_{AB} = 24.727 \times 10^3 \text{ N}$$

$$d^2 = \frac{2F_{AB}}{\pi\tau} = \frac{(2)(24.727 \times 10^3)}{\pi(120 \times 10^6)} = 131.181 \times 10^{-6} \text{ m}^2$$

$$d = 11.4534 \times 10^{-3} \text{ m} \qquad 11.45 \text{ mm} \blacktriangleleft$$

(*b*) Bearing stress in *AB* at *A*.

$$A_b = td = (0.016)(11.4534 \times 10^{-3}) = 183.254 \times 10^{-6} \text{ m}^2$$

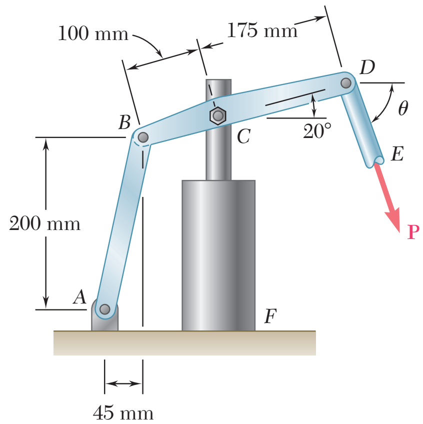
$$\sigma_b = \frac{F_{AB}}{A_b} = \frac{24.727 \times 10^3}{183.254 \times 10^{-6}} = 134.933 \times 10^6 \text{ Pa} \qquad 134.9 \text{ MPa} \blacktriangleleft$$

(*c*) Bearing stress in support brackets at *B*.

$$A = td = (0.012)(11.4534 \times 10^{-3}) = 137.441 \times 10^{-6} \text{ m}^2$$

$$\sigma_b = \frac{\frac{1}{2}F_{AB}}{A} = \frac{(0.5)(24.727 \times 10^3)}{137.441 \times 10^{-6}} = 89.955 \times 10^6 \text{ Pa} \qquad 90.0 \text{ MPa} \blacktriangleleft$$

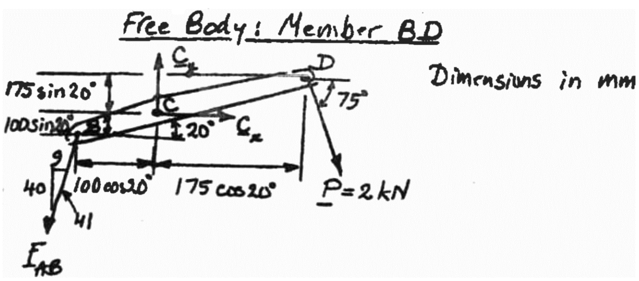
PROBLEM 1.26



The hydraulic cylinder CF, which partially controls the position of rod DE, has been locked in the position shown. Member BD is 15 mm thick and is connected at C to the vertical rod by a 9-mm-diameter bolt. Knowing that $P = 2$ kN and $\theta = 75°$, determine (*a*) the average shearing stress in the bolt, (*b*) the bearing stress at C in member BD.

SOLUTION

Free Body: Member BD.



$$+\circlearrowleft \Sigma M_c = 0: \quad \frac{40}{41}F_{AB}(100\cos 20°) - \frac{9}{4}F_{AB}(100\sin 20°)$$

$$-(2\text{ kN})\cos 75°(175\sin 20°) - (2\text{ kN})\sin 75°(175\cos 20°) = 0$$

$$\frac{100}{41}F_{AB}(40\cos 20° - 9\sin 20°) = (2\text{ kN})(175)\sin(75° + 20°)$$

$$F_{AB} = 4.1424\text{ kN}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - \frac{9}{41}(4.1424\text{ kN}) + (2\text{ kN})\cos 75° = 0$$

$$C_x = 0.39167\text{ kN}$$

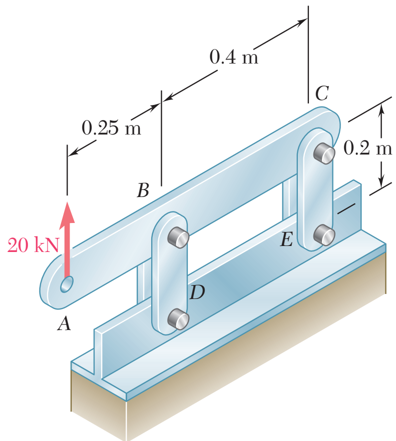
$$+\uparrow \Sigma F_y = 0: \quad C_y - \frac{40}{41}(4.1424\text{ kN}) - (2\text{ kN})\sin 75° = 0$$

$$C_y = 5.9732\text{ kN}$$

$$\mathbf{C} = 5.9860\text{ kN} \measuredangle 86.2°$$

(*a*) $$\tau_{\text{ave}} = \frac{C}{A} = \frac{5.9860 \times 10^3\text{ N}}{\pi(0.0045\text{ m})^2} = 94.1 \times 10^6\text{ Pa} = 94.1\text{ MPa}$$ ◀

(*b*) $$\tau_b = \frac{C}{td} = \frac{5.9860 \times 10^3\text{ N}}{(0.015\text{ m})(0.009\text{ m})} = 44.3 \times 10^6\text{ Pa} = 44.3\text{ MPa}$$ ◀



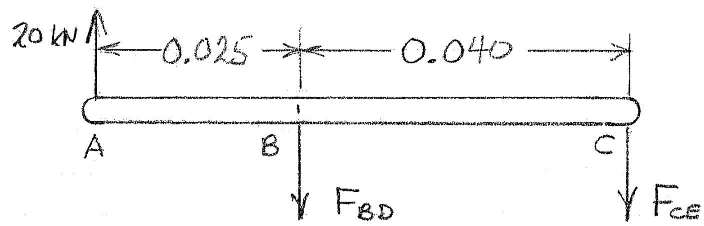
PROBLEM 1.27

For the assembly and loading of Prob. 1.7, determine (*a*) the average shearing stress in the pin at *B*, (*b*) the average bearing stress at *B* in member *BD*, (*c*) the average bearing stress at *B* in member *ABC*, knowing that this member has a 10 × 50-mm uniform rectangular cross section.

PROBLEM 1.7 Each of the four vertical links has an 8 × 36-mm uniform rectangular cross section and each of the four pins has a 16-mm diameter. Determine the maximum value of the average normal stress in the links connecting (*a*) points *B* and *D*, (*b*) points *C* and *E*.

SOLUTION

Use bar *ABC* as a free body.



$$+\circlearrowleft \Sigma M_C = 0: \quad (0.040)F_{BD} - (0.025 + 0.040)(20 \times 10^3) = 0$$

$$F_{BD} = 32.5 \times 10^3 \text{N}$$

(*a*) Shear pin at *B*. $\tau = \dfrac{F_{BD}}{2A}$ for double shear

where $A = \dfrac{\pi}{4}d^2 = \dfrac{\pi}{4}(0.016)^2 = 201.06 \times 10^{-6} \text{m}^2$

$$\tau = \frac{32.5 \times 10^3}{(2)(201.06 \times 10^{-6})} = 80.822 \times 10^6 \text{ Pa}$$

$\tau = 80.8 \text{ MPa}$ ◀

(*b*) Bearing: link *BD*. $A = dt = (0.016)(0.008) = 128 \times 10^{-6} \text{m}^2$

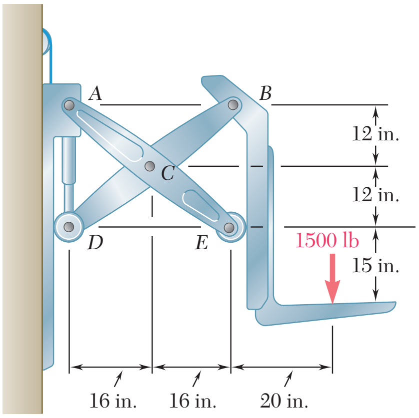
$$\sigma_b = \frac{\frac{1}{2}F_{BD}}{A} = \frac{(0.5)(32.5 \times 10^3)}{128 \times 10^{-6}} = 126.95 \times 10^6 \text{ Pa}$$

$\sigma_b = 127.0 \text{ MPa}$ ◀

(*c*) Bearing in *ABC* at *B*. $A = dt = (0.016)(0.010) = 160 \times 10^{-6} \text{m}^2$

$$\sigma_b = \frac{F_{BD}}{A} = \frac{32.5 \times 10^3}{160 \times 10^{-6}} = 203.12 \times 10^6 \text{ Pa}$$

$\sigma_b = 203 \text{ MPa}$ ◀

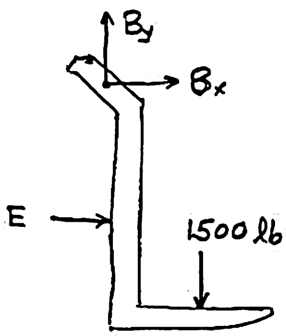


PROBLEM 1.28

Two identical linkage-and-hydraulic-cylinder systems control the position of the forks of a fork-lift truck. The load supported by the one system shown is 1500 lb. Knowing that the thickness of member *BD* is $\frac{5}{8}$ in., determine (*a*) the average shearing stress in the $\frac{1}{2}$-in.-diameter pin at *B*, (*b*) the bearing stress at *B* in member *BD*.

SOLUTION

Use one fork as a free body.



$$+\circlearrowleft \Sigma M_B = 0: \quad 24E - (20)(1500) = 0$$

$$E = 1250 \text{ lb} \longrightarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad E + B_x = 0$$

$$B_x = -E$$

$$B_x = 1250 \text{ lb} \longleftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad B_y - 1500 = 0 \qquad B_y = 1500 \text{ lb}$$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{1250^2 + 1500^2} = 1952.56 \text{ lb}$$

(*a*) Shearing stress in pin at *B*.

$$A_{\text{pin}} = \frac{\pi}{4} d_{\text{pin}}^2 = \frac{\pi}{4}\left(\frac{1}{2}\right)^2 = 0.196350 \text{ in}^2$$

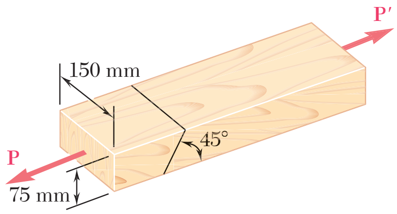
$$\tau = \frac{B}{A_{\text{pin}}} = \frac{1952.56}{0.196350} = 9.94 \times 10^3 \text{ psi}$$

$\tau = 9.94$ ksi ◀

(*b*) Bearing stress at *B*.

$$\sigma = \frac{B}{dt} = \frac{1952.56}{\left(\frac{1}{2}\right)\left(\frac{5}{8}\right)} = 6.25 \times 10^3 \text{ psi}$$

$\sigma = 6.25$ ksi ◀



PROBLEM 1.29

Two wooden members of uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $P = 11$ kN, determine the normal and shearing stresses in the glued splice.

SOLUTION

$$\theta = 90° - 45° = 45°$$

$$P = 11 \text{ kN} = 11 \times 10^3 \text{ N}$$

$$A_0 = (150)(75) = 11.25 \times 10^3 \text{ mm}^2 = 11.25 \times 10^{-3} \text{ m}^2$$

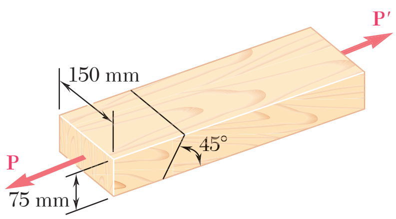
$$\sigma = \frac{P\cos^2\theta}{A_0} = \frac{(11 \times 10^3)\cos^2 45°}{11.25 \times 10^{-3}} = 489 \times 10^3 \text{ Pa}$$

$\sigma = 489$ kPa ◀

$$\tau = \frac{P\sin 2\theta}{2A_0} = \frac{(11 \times 10^3)(\sin 90°)}{(2)(11.25 \times 10^{-3})} = 489 \times 10^3 \text{ Pa}$$

$\tau = 489$ kPa ◀

PROBLEM 1.30



Two wooden members of uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that the maximum allowable shearing stress in the glued splice is 620 kPa, determine (*a*) the largest load **P** that can be safely applied, (*b*) the corresponding tensile stress in the splice.

SOLUTION

$$\theta = 90° - 45° = 45°$$

$$A_0 = (150)(75) = 11.25 \times 10^3 \text{mm}^2 = 11.25 \times 10^{-3} \text{m}^2$$

$$\tau = 620 \text{ kPa} = 620 \times 10^3 \text{Pa}$$

$$\tau = \frac{P \sin 2\theta}{2A_0}$$

(*a*) $$P = \frac{2A_0\tau}{\sin 2\theta} = \frac{(2)(11.25 \times 10^{-3})(620 \times 10^3)}{\sin 90°}$$

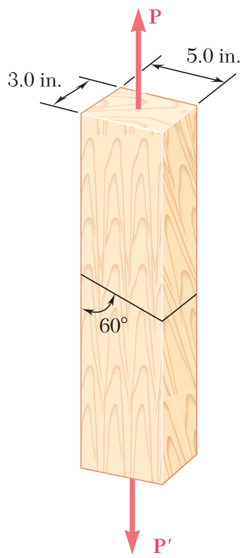
$$= 13.95 \times 10^3 \text{ N}$$

$P = 13.95$ kN ◀

(*b*) $$\sigma = \frac{P\cos^2\theta}{A_0} = \frac{(13.95 \times 10^3)(\cos 45°)^2}{11.25 \times 10^{-3}}$$

$$= 620 \times 10^3 \text{ Pa}$$

$\sigma = 620$ kPa ◀



PROBLEM 1.31

The 1.4-kip load **P** is supported by two wooden members of uniform cross section that are joined by the simple glued scarf splice shown. Determine the normal and shearing stresses in the glued splice.

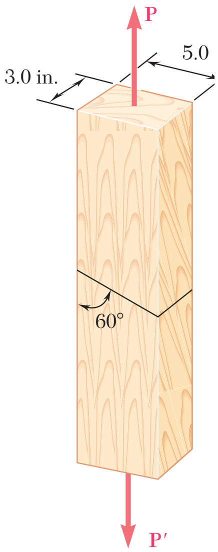
SOLUTION

$$P = 1400 \text{ lb} \qquad \theta = 90° - 60° = 30°$$

$$A_0 = (5.0)(3.0) = 15 \text{ in}^2$$

$$\sigma = \frac{P\cos^2\theta}{A_0} = \frac{(1400)(\cos 30°)^2}{15} \qquad \sigma = 70.0 \text{ psi} ◀$$

$$\tau = \frac{P\sin 2\theta}{2A_0} = \frac{(1400)\sin 60°}{(2)(15)} \qquad \tau = 40.4 \text{ psi} ◀$$



PROBLEM 1.32

Two wooden members of uniform cross section are joined by the simple scarf splice shown. Knowing that the maximum allowable tensile stress in the glued splice is 75 psi, determine (*a*) the largest load **P** that can be safely supported, (*b*) the corresponding shearing stress in the splice.

SOLUTION

$$A_0 = (5.0)(3.0) = 15 \text{ in}^2$$

$$\theta = 90° - 60° = 30°$$

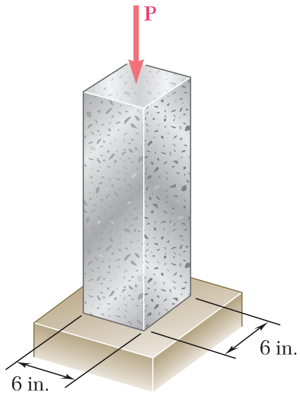
$$\sigma = \frac{P\cos^2\theta}{A_0}$$

(*a*)
$$P = \frac{\sigma A_0}{\cos^2\theta} = \frac{(75)(15)}{\cos^2 30°} = 1500 \text{ lb}$$

$P = 1.500$ kips ◀

(*b*)
$$\tau = \frac{P\sin 2\theta}{2A_0} = \frac{(1500)\sin 60°}{(2)(15)}$$

$\tau = 43.3$ psi ◀



PROBLEM 1.33

A centric load **P** is applied to the granite block shown. Knowing that the resulting maximum value of the shearing stress in the block is 2.5 ksi, determine (*a*) the magnitude of **P**, (*b*) the orientation of the surface on which the maximum shearing stress occurs, (*c*) the normal stress exerted on that surface, (*d*) the maximum value of the normal stress in the block.

SOLUTION

$$A_0 = (6)(6) = 36 \text{ in}^2$$

$$\tau_{max} = 2.5 \text{ ksi}$$

$$\theta = 45° \text{ for plane of } \tau_{max}$$

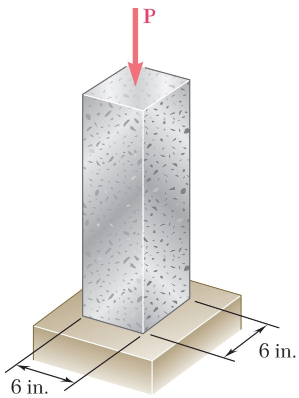
(*a*) $\tau_{max} = \dfrac{|P|}{2A_0} \quad \therefore \quad |P| = 2A_0\tau_{max} = (2)(36)(2.5)$ $\qquad P = 180.0 \text{ kips}$ ◀

(*b*) $\sin 2\theta = 1 \quad 2\theta = 90°$ $\qquad \theta = 45.0°$ ◀

(*c*) $\sigma_{45} = \dfrac{P}{A_0}\cos^2 45° = \dfrac{P}{2A_0} = -\dfrac{180}{(2)(36)}$ $\qquad \sigma_{45} = -2.50 \text{ ksi}$ ◀

(*d*) $\sigma_{max} = \dfrac{P}{A_0} = \dfrac{-180}{36}$ $\qquad \sigma_{max} = -5.00 \text{ ksi}$ ◀

PROBLEM 1.34



A 240-kip load **P** is applied to the granite block shown. Determine the resulting maximum value of (*a*) the normal stress, (*b*) the shearing stress. Specify the orientation of the plane on which each of these maximum values occurs.

SOLUTION

$$A_0 = (6)(6) = 36 \text{ in}^2$$

$$\sigma = \frac{P}{A_0}\cos^2\theta = \frac{-240}{36}\cos^2\theta = -6.67\cos^2\theta$$

(*a*) max tensile stress = 0 at $\theta = 90.0°$

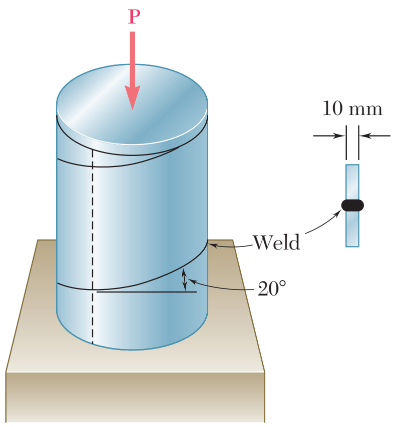
max. compressive stress = 6.67 ksi at $\theta = 0°$ ◀

(*b*) $\tau_{max} = \frac{P}{2A_0} = \frac{240}{(2)(36)}$

$\tau_{max} = 3.33$ ksi ◀

at $\theta = 45°$

PROBLEM 1.35



A steel pipe of 400-mm outer diameter is fabricated from 10-mm thick plate by welding along a helix that forms an angle of 20° with a plane perpendicular to the axis of the pipe. Knowing that a 300-kN axial force **P** is applied to the pipe, determine the normal and shearing stresses in directions respectively normal and tangential to the weld.

SOLUTION

$$d_o = 0.400 \text{ m}$$

$$r_o = \frac{1}{2} d_o = 0.200 \text{ m}$$

$$r_i = r_o - t = 0.200 - 0.010 = 0.190 \text{ m}$$

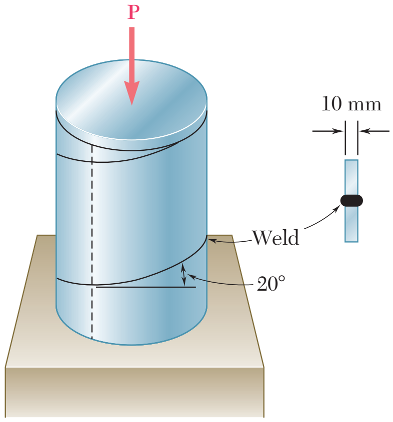
$$A_o = \pi(r_o^2 - r_i^2) = \pi(0.200^2 - 0.190^2)$$

$$= 12.2522 \times 10^{-3} \text{ m}^2$$

$$\theta = 20°$$

$$\sigma = \frac{P}{A_o} = \cos^2\theta = \frac{-300 \times 10^3 \cos^2 20°}{12.2522 \times 10^{-3}} = 21.621 \times 10^6 \text{ Pa} \qquad \sigma = -21.6 \text{ MPa} \blacktriangleleft$$

$$\tau = \frac{P}{2A_0} = \sin 2\theta = \frac{-300 \times 10^3 \sin 40°}{(2)(12.2522 \times 10^{-3})} = 7.8695 \times 10^6 \text{ Pa} \qquad \tau = 7.87 \text{ MPa} \blacktriangleleft$$



PROBLEM 1.36

A steel pipe of 400-mm outer diameter is fabricated from 10-mm thick plate by welding along a helix that forms an angle of 20° with a plane perpendicular to the axis of the pipe. Knowing that the maximum allowable normal and shearing stresses in the directions respectively normal and tangential to the weld are $\sigma = 60$ MPa and $\tau = 36$ MPa, determine the magnitude P of the largest axial force that can be applied to the pipe.

SOLUTION

$$d_o = 0.400 \text{ m}$$

$$r_o = \frac{1}{2} d_o = 0.200 \text{ m}$$

$$r_i = r_o - t = 0.200 - 0.010 = 0.190 \text{ m}$$

$$A_o = \pi (r_o^2 - r_i^2) = \pi (0.200^2 - 0.190^2)$$

$$= 12.2522 \times 10^{-3} \text{ m}^2$$

$$\theta = 20°$$

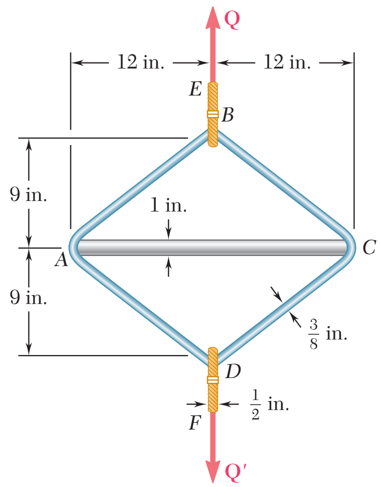
Based on $|\sigma| = 60$ MPa: $\sigma = \dfrac{P}{A_0} \cos^2 \theta$

$$P = \frac{A_o \sigma}{\cos^2 \theta} = \frac{(12.2522 \times 10^{-3})(60 \times 10^6)}{\cos^2 20°} = 832.52 \times 10^3 \text{ N}$$

Based on $|\tau| = 30$ MPa: $\tau = \dfrac{P}{2A_o} \sin 2\theta$

$$P = \frac{2 A_o \tau}{\sin 2\theta} = \frac{(2)(12.2522 \times 10^{-3})(36 \times 10^6)}{\sin 40°} = 1372.39 \times 10^3 \text{ N}$$

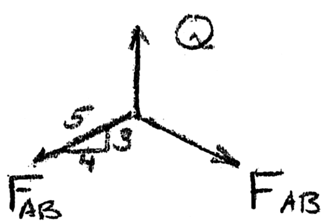
Smaller value is the allowable value of P. $P = 833$ kN ◀



PROBLEM 1.37

A steel loop *ABCD* of length 5 ft and of $\frac{3}{8}$-in. diameter is placed as shown around a 1-in.-diameter aluminum rod *AC*. Cables *BE* and *DF*, each of $\frac{1}{2}$-in. diameter, are used to apply the load **Q**. Knowing that the ultimate strength of the steel used for the loop and the cables is 70 ksi, and that the ultimate strength of the aluminum used for the rod is 38 ksi, determine the largest load **Q** that can be applied if an overall factor of safety of 3 is desired.

SOLUTION



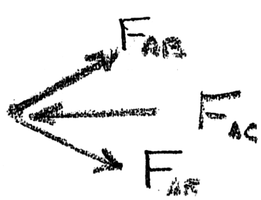
Using joint *B* as a free body and considering symmetry,

$$2 \cdot \frac{3}{5} F_{AB} - Q = 0 \qquad Q = \frac{6}{5} F_{AB}$$

Using joint *A* as a free body and considering symmetry,

$$2 \cdot \frac{4}{5} F_{AB} - F_{AC} = 0$$

$$\frac{8}{5} \cdot \frac{5}{6} Q - F_{AC} = 0 \quad \therefore \quad Q = \frac{3}{4} F_{AC}$$



Based on strength of cable *BE*,

$$Q_U = \sigma_U A = \sigma_U \frac{\pi}{4} d^2 = (70) \frac{\pi}{4} \left(\frac{1}{2}\right)^2 = 13.7445 \text{ kips}$$

Based on strength of steel loop,

$$Q_U = \frac{6}{5} F_{AB,U} = \frac{6}{5} \sigma_U A = \frac{6}{5} \sigma_U \frac{\pi}{4} d^2$$

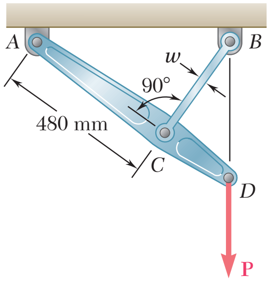
$$= \frac{6}{5} (70) \frac{\pi}{4} \left(\frac{3}{8}\right)^2 = 9.2775 \text{ kips}$$

Based on strength of rod *AC*,

$$Q_U = \frac{3}{4} F_{AC,U} = \frac{3}{4} \sigma_U A = \frac{3}{4} \sigma_U \frac{\pi}{4} d^2 = \frac{3}{4} (38) \frac{\pi}{4} (1.0)^2 = 22.384 \text{ kips}$$

Actual ultimate load Q_U is the smallest, $\therefore \quad Q_U = 9.2775$ kips

Allowable load: $\quad Q = \frac{Q_U}{F.S.} = \frac{9.2775}{3} = 3.0925$ kips $\qquad Q = 3.09$ kips ◀

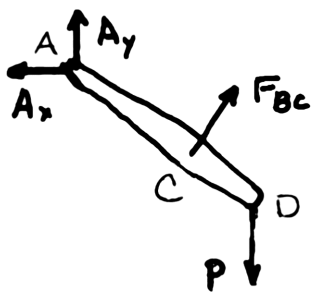


PROBLEM 1.38

Link *BC* is 6 mm thick, has a width $w = 25$ mm, and is made of a steel with a 480-MPa ultimate strength in tension. What was the safety factor used if the structure shown was designed to support a 16-kN load **P**?

SOLUTION

Use bar *ACD* as a free body and note that member *BC* is a two-force member.



$$\Sigma M_A = 0:$$

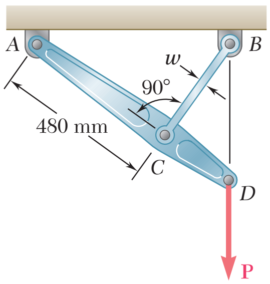
$$(480)F_{BC} - (600)P = 0$$

$$F_{BC} = \frac{600}{480}P = \frac{(600)(16 \times 10^3)}{480} = 20 \times 10^3 \text{ N}$$

Ultimate load for member *BC*: $F_U = \sigma_U A$

$$F_U = (480 \times 10^6)(0.006)(0.025) = 72 \times 10^3 \text{ N}$$

Factor of safety: $\text{F.S.} = \dfrac{F_U}{F_{BC}} = \dfrac{72 \times 10^3}{20 \times 10^3}$ $\qquad$ F.S. = 3.60 ◀

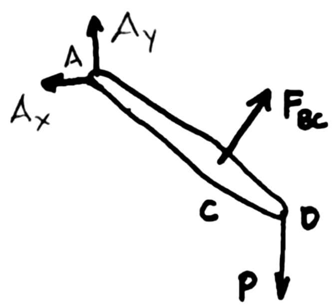


PROBLEM 1.39

Link *BC* is 6 mm thick and is made of a steel with a 450-MPa ultimate strength in tension. What should be its width *w* if the structure shown is being designed to support a 20-kN load **P** with a factor of safety of 3?

SOLUTION

Use bar *ACD* as a free body and note that member *BC* is a two-force member.



$$\Sigma M_A = 0:$$

$$(480)F_{BC} - 600P = 0$$

$$F_{BC} = \frac{600P}{480} = \frac{(600)(20\times 10^3)}{480} = 25\times 10^3 \text{ N}$$

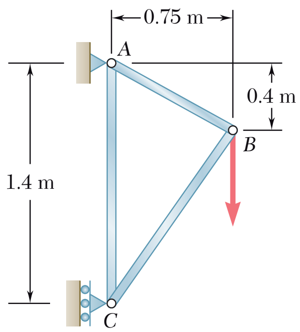
For a factor of safety F.S. = 3, the ultimate load of member *BC* is

$$F_U = (\text{F.S.})(F_{BC}) = (3)(25\times 10^3) = 75\times 10^3 \text{ N}$$

But $F_U = \sigma_U A \quad \therefore \quad A = \frac{F_U}{\sigma_U} = \frac{75\times 10^3}{450\times 10^6} = 166.667\times 10^{-6}\text{ m}^2$

For a rectangular section, $A = wt$ or $w = \frac{A}{t} = \frac{166.667\times 10^{-6}}{0.006} = 27.778\times 10^{-3}\text{ m}$

$w = 27.8 \text{ mm}$ ◀



PROBLEM 1.40

Members *AB* and *BC* of the truss shown are made of the same alloy. It is known that a 20-mm-square bar of the same alloy was tested to failure and that an ultimate load of 120 kN was recorded. If a factor of safety of 3.2 is to be achieved for both bars, determine the required cross-sectional area of (*a*) bar *AB*, (*b*) bar *AC*.

SOLUTION

Length of member *AB*:

$$\ell_{AB} = \sqrt{0.75^2 + 0.4^2} = 0.85 \text{ m}$$

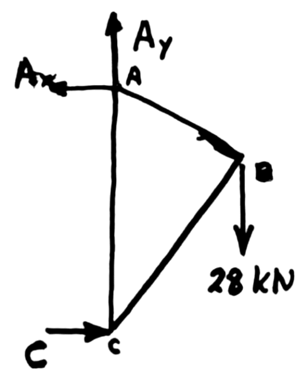
Use entire truss as a free body.

$$+\circlearrowleft \Sigma M_c = 0: \quad 1.4A_x - (0.75)(28) = 0$$

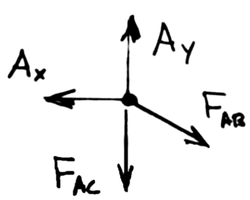
$$A_x = 15 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 28 = 0$$

$$A_y = 28 \text{ kN}$$



Use Joint *A* as free body.



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{0.75}{0.85} F_{AB} - A_x = 0$$

$$F_{AB} = \frac{(0.85)(15)}{0.75} = 17 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - F_{AC} - \frac{0.4}{0.85} F_{AB} = 0$$

$$F_{AC} = 28 - \frac{(0.4)(17)}{0.85} = 20 \text{ kN}$$

For the test bar, $A = (0.020)^2 = 400 \times 10^{-6} \text{ m}^2 \quad P_U = 120 \times 10^3 \text{ N}$

For the material, $\sigma_U = \dfrac{P_U}{A} = \dfrac{120 \times 10^3}{400 \times 10^{-6}} = 300 \times 10^6 \text{ Pa}$

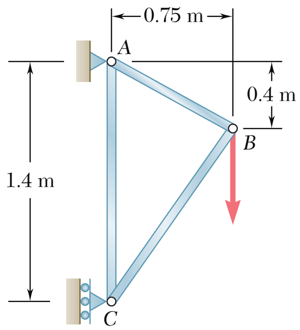
PROBLEM 1.40 *(Continued)*

(*a*) For member *AB*: $\text{F.S.} = \dfrac{P_U}{F_{AB}} = \dfrac{\sigma_U A_{AB}}{F_{AB}}$

$$A_{AB} = \frac{(\text{F.S.})F_{AB}}{\sigma_U} = \frac{(3.2)(17\times 10^3)}{300\times 10^6} = 181.333\times 10^{-6}\ \text{m}^2 \qquad A_{AB} = 181.3\ \text{mm}^2 \blacktriangleleft$$

(*b*) For member *AC*: $\text{F.S.} = \dfrac{P_U}{F_{AC}} = \dfrac{\sigma_U A_{AC}}{F_{AC}}$

$$A_{AC} = \frac{(\text{F.S.})F_{AC}}{\sigma_U} = \frac{(3.2)(20\times 10^3)}{300\times 10^6} = 213.33\times 10^{-6}\ \text{m}^2 \qquad A_{AC} = 213\ \text{mm}^2 \blacktriangleleft$$



PROBLEM 1.41

Members AB and BC of the truss shown are made of the same alloy. It is known that a 20-mm-square bar of the same alloy was tested to failure and that an ultimate load of 120 kN was recorded. If bar AB has a cross-sectional area of 225 mm^2, determine (*a*) the factor of safety for bar AB and (*b*) the cross-sectional area of bar AC if it is to have the same factor of safety as bar AB.

SOLUTION

Length of member AB:

$$\ell_{AB} = \sqrt{0.75^2 + 0.4^2} = 0.85 \text{ m}$$

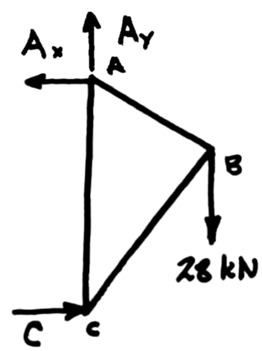
Use entire truss as a free body.

$$+\circlearrowleft \Sigma M_c = 0: \quad 1.4A_x - (0.75)(28) = 0$$

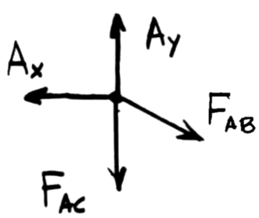
$$A_x = 15 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 28 = 0$$

$$A_y = 28 \text{ kN}$$



Use Joint A as free body.



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{0.75}{0.85}F_{AB} - A_x = 0$$

$$F_{AB} = \frac{(0.85)(15)}{0.75} = 17 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - F_{AC} - \frac{0.4}{0.85}F_{AB} = 0$$

$$F_{AC} = 28 - \frac{(0.4)(17)}{0.85} = 20 \text{ kN}$$

For the test bar, $\quad A = (0.020)^2 = 400 \times 10^{-6} \text{ m}^2 \quad P_U = 120 \times 10^3 \text{ N}$

For the material, $\quad \sigma_U = \frac{P_U}{A} = \frac{120 \times 10^3}{400 \times 10^{-6}} = 300 \times 10^6 \text{ Pa}$

PROBLEM 1.41 *(Continued)*

(*a*) For bar *AB*:

$$\text{F.S.} = \frac{F_U}{F_{AB}} = \frac{\sigma_U A_{AB}}{F_{AB}} = \frac{(300 \times 10^6)(225 \times 10^{-6})}{17 \times 10^3}$$

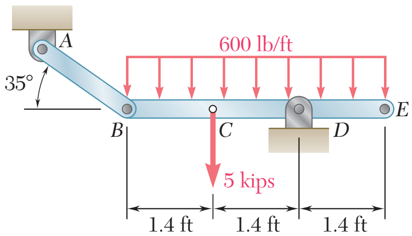
$$\text{F.S.} = 3.97 \blacktriangleleft$$

(*b*) For bar *AC*:

$$\text{F.S.} = \frac{F_U}{F_{AC}} = \frac{\sigma_U A_{AC}}{F_{AC}}$$

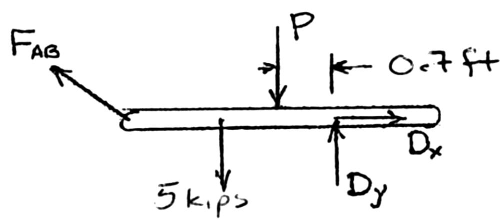
$$A_{AC} = \frac{(\text{F.S.})F_{AC}}{\sigma_U} = \frac{(3.97)(20 \times 10^3)}{300 \times 10^6} = 264.67 \times 10^{-6}\ \text{m}^2 \qquad A_{AC} = 265\ \text{mm}^2 \blacktriangleleft$$

PROBLEM 1.42



Link AB is to be made of a steel for which the ultimate normal stress is 65 ksi. Determine the cross-sectional area of AB for which the factor of safety will be 3.20. Assume that the link will be adequately reinforced around the pins at A and B.

SOLUTION



$$P = (4.2)(0.6) = 2.52 \text{ kips}$$

$$+\circlearrowleft \Sigma M_D = 0: \quad -(2.8)(F_{AB}\sin 35°) + (0.7)(2.52) + (1.4)(5) = 0$$

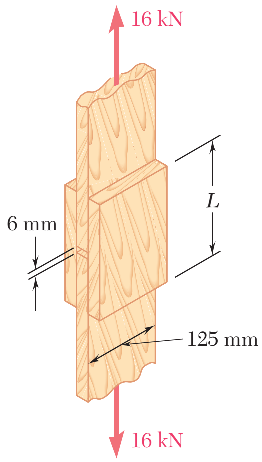
$$F_{AB} = 5.4570 \text{ kips}$$

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{\sigma_{\text{ult}}}{F.S.}$$

$$A_{AB} = \frac{(F.S.)F_{AB}}{\sigma_{\text{ult}}} = \frac{(3.20)(5.4570 \text{ kips})}{65 \text{ ksi}}$$

$$= 0.26854 \text{ in}^2$$

$$A_{AB} = 0.268 \text{ in}^2 \blacktriangleleft$$



PROBLEM 1.43

Two wooden members are joined by plywood splice plates that are fully glued on the contact surfaces. Knowing that the clearance between the ends of the members is 6 mm and that the ultimate shearing stress in the glued joint is 2.5 MPa, determine the length L for which the factor of safety is 2.75 for the loading shown.

SOLUTION

$$\tau_{\text{all}} = \frac{2.5\text{ MPa}}{2.75} = 0.90909\text{ MPa}$$

On one face of the upper contact surface,

$$A = \frac{L - 0.006\text{ m}}{2}(0.125\text{ m})$$

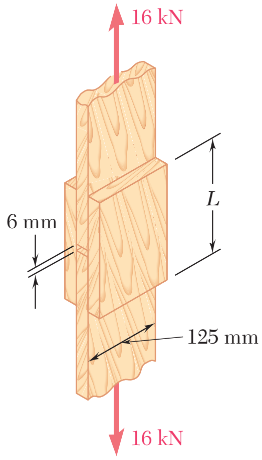
Since there are 2 contact surfaces,

$$\tau_{\text{all}} = \frac{P}{2A}$$

$$0.90909 \times 10^6 = \frac{16 \times 10^3}{(L - 0.006)(0.125)}$$

$$L = 0.14680\text{ m}$$

146.8 mm ◀



PROBLEM 1.44

For the joint and loading of Prob. 1.43, determine the factor of safety when $L = 180$ mm.

PROBLEM 1.43 Two wooden members are joined by plywood splice plates that are fully glued on the contact surfaces. Knowing that the clearance between the ends of the members is 6 mm and that the ultimate shearing stress in the glued joint is 2.5 MPa, determine the length L for which the factor of safety is 2.75 for the loading shown.

SOLUTION

Area of one face of upper contact surface:

$$A = \frac{0.180 \text{ m} - 0.006 \text{ m}}{2}(0.125 \text{ m})$$

$$A = 10.8750 \times 10^{-3} \text{ m}^2$$

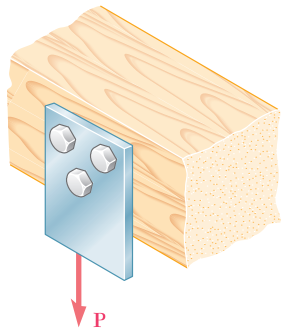
Since there are two surfaces,

$$\tau_{\text{all}} = \frac{P}{2A} = \frac{16 \times 10^3 \text{ N}}{2(10.8750 \times 10^{-3} \text{ m}^2)}$$

$$\tau_{\text{all}} = 0.73563 \text{ MPa}$$

$$\text{F.S.} = \frac{\tau_u}{\tau_{\text{all}}} = \frac{2.5 \text{ MPa}}{0.73563 \text{ MPa}} = 3.40 \quad \blacktriangleleft$$

PROBLEM 1.45



Three $\frac{3}{4}$-in.-diameter steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a load $P = 24$ kips and that the ultimate shearing stress for the steel used is 52 ksi, determine the factor of safety for this design.

SOLUTION

For each bolt,

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}\left(\frac{3}{4}\right)^2 = 0.44179 \text{ in}^2$$

$$P_U = A\tau_U = (0.44179)(52)$$

$$= 22.973 \text{ kips}$$

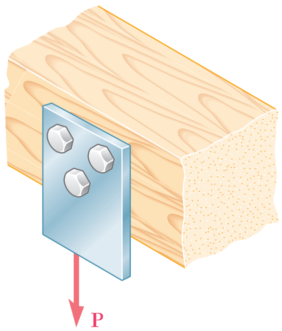
For the three bolts,

$$P_U = (3)(22.973) = 68.919 \text{ kips}$$

Factor of safety:

$$F.S. = \frac{P_U}{P} = \frac{68.919}{24}$$

$F.S. = 2.87$ ◀



PROBLEM 1.46

Three steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a load $P = 28$ kips, that the ultimate shearing stress for the steel used is 52 ksi, and that a factor of safety of 3.25 is desired, determine the required diameter of the bolts.

SOLUTION

For each bolt, $P = \frac{24}{3} = 8$ kips

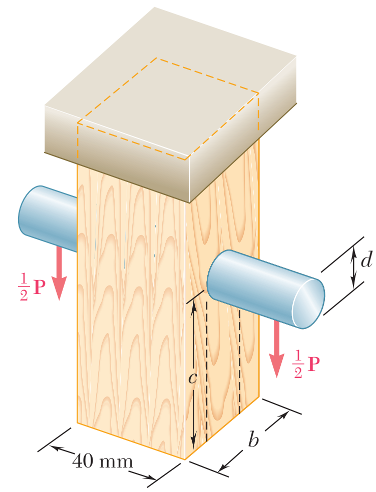
Required: $P_U = (F.S.)P = (3.25)(8.0) = 26.0$ kips

$$\tau_U = \frac{P_U}{A} = \frac{P_U}{\frac{\pi}{4}d^2} = \frac{4P_U}{\pi d^2}$$

$$d = \sqrt{\frac{4P_U}{\pi\tau_U}} = \sqrt{\frac{(4)(26.0)}{\pi(52)}} = 0.79789 \text{ in.}$$

$d = 0.798$ in. ◀

PROBLEM 1.47



A load **P** is supported as shown by a steel pin that has been inserted in a short wooden member hanging from the ceiling. The ultimate strength of the wood used is 60 MPa in tension and 7.5 MPa in shear, while the ultimate strength of the steel is 145 MPa in shear. Knowing that $b = 40$ mm, $c = 55$ mm, and $d = 12$ mm, determine the load **P** if an overall factor of safety of 3.2 is desired.

SOLUTION

Based on double shear in pin,

$$P_U = 2A\tau_U = 2\frac{\pi}{4}d^2\tau_U$$

$$= \frac{\pi}{4}(2)(0.012)^2(145\times10^6) = 32.80\times10^3\text{ N}$$

Based on tension in wood,

$$P_U = A\sigma_U = w(b-d)\sigma_U$$

$$= (0.040)(0.040-0.012)(60\times10^6)$$

$$= 67.2\times10^3\text{ N}$$

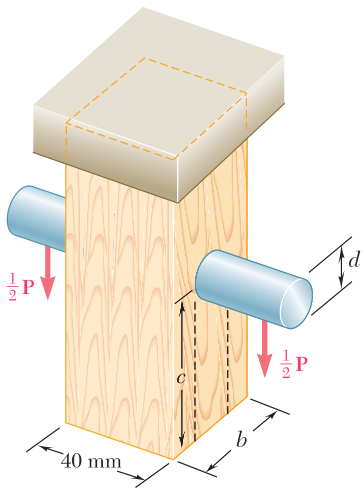
Based on double shear in the wood,

$$P_U = 2A\tau_U = 2wc\tau_U = (2)(0.040)(0.055)(7.5\times10^6)$$

$$= 33.0\times10^3\text{ N}$$

Use smallest $P_U = 32.8\times10^3\text{ N}$

Allowable: $P = \dfrac{P_U}{F.S.} = \dfrac{32.8\times10^3}{3.2} = 10.25\times10^3\text{ N}$ 10.25 kN ◀



PROBLEM 1.48

For the support of Prob. 1.47, knowing that the diameter of the pin is $d = 16$ mm and that the magnitude of the load is $P = 20$ kN, determine (*a*) the factor of safety for the pin, (*b*) the required values of *b* and *c* if the factor of safety for the wooden members is the same as that found in part *a* for the pin.

PROBLEM 1.47 A load **P** is supported as shown by a steel pin that has been inserted in a short wooden member hanging from the ceiling. The ultimate strength of the wood used is 60 MPa in tension and 7.5 MPa in shear, while the ultimate strength of the steel is 145 MPa in shear. Knowing that $b = 40$ mm, $c = 55$ mm, and $d = 12$ mm, determine the load **P** if an overall factor of safety of 3.2 is desired.

SOLUTION

$$P = 20 \text{ kN} = 20 \times 10^3 \text{ N}$$

(*a*) Pin:

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.016)^2 = 2.01.06 \times 10^{-6} \text{ m}^2$$

Double shear:

$$\tau = \frac{P}{2A} \qquad \tau_U = \frac{P_U}{2A}$$

$$P_U = 2A\tau_U = (2)(201.16 \times 10^{-6})(145 \times 10^6) = 58.336 \times 10^3 \text{ N}$$

$$F.S. = \frac{P_U}{P} = \frac{58.336 \times 10^3}{20 \times 10^3}$$

$F.S. = 2.92$ ◀

(*b*) Tension in wood: $P_U = 58.336 \times 10^3$ N for same F.S.

$$\sigma_U = \frac{P_U}{A} = \frac{P_U}{w(b-d)} \quad \text{where} \quad w = 40 \text{ mm} = 0.040 \text{ m}$$

$$b = d + \frac{P_U}{w\sigma_U} = 0.016 + \frac{58.336 \times 10^3}{(0.040)(60 \times 10^6)} = 40.3 \times 10^{-3} \text{ m}$$

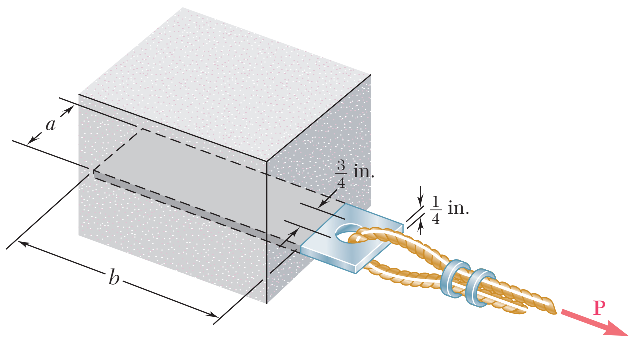
$b = 40.3$ mm ◀

Shear in wood: $P_U = 58.336 \times 10^3$ N for same F.S.

Double shear: each area is $A = wc$ $\qquad \tau_U = \frac{P_U}{2A} = \frac{P_U}{2wc}$

$$c = \frac{P_U}{2w\tau_U} = \frac{58.336 \times 10^3}{(2)(0.040)(7.5 \times 10^6)} = 97.2 \times 10^{-3} \text{ m}$$

$c = 97.2$ mm ◀

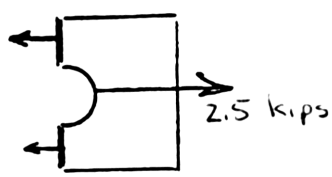


PROBLEM 1.49

A steel plate $\frac{1}{4}$ in. thick is embedded in a concrete wall to anchor a high-strength cable as shown. The diameter of the hole in the plate is $\frac{3}{4}$ in., the ultimate strength of the steel used is 36 ksi, and the ultimate bonding stress between plate and concrete is 300 psi. Knowing that a factor of safety of 3.60 is desired when $P = 2.5$ kips, determine (*a*) the required width *a* of the plate, (*b*) the minimum depth *b* to which a plate of that width should be embedded in the concrete slab. (Neglect the normal stresses between the concrete and the end of the plate.)

SOLUTION

Based on tension in plate,



$$A = (a - d)t$$

$$P_U = \sigma_U A$$

$$F.S. = \frac{P_U}{P} = \frac{\sigma_U (a - d)t}{P}$$

Solving for *a*,

$$a = d + \frac{(F.S.)P}{\sigma_U t} = \frac{3}{4} + \frac{(3.60)(2.5)}{(36)\left(\frac{1}{4}\right)}$$

(*a*) $a = 1.750$ in. ◀

Based on shear between plate and concrete slab,

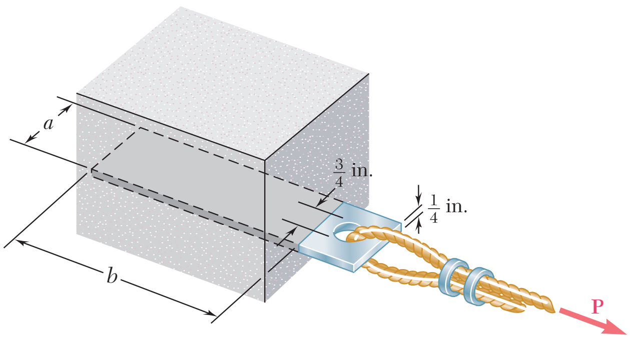
$$A = \text{perimeter} \times \text{depth} = 2(a + t)b \qquad \tau_U = 0.300 \text{ ksi}$$

$$P_U = \tau_U A = 2\tau_U (a + t)b \qquad F.S. = \frac{P_U}{P}$$

Solving for *b*,

$$b = \frac{(F.S.)P}{2(a + t)\tau_U} = \frac{(3.6)(2.5)}{(2)\left(1.75 + \frac{1}{4}\right)(0.300)}$$

(*b*) $b = 7.50$ in. ◀



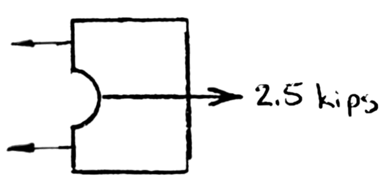
PROBLEM 1.50

Determine the factor of safety for the cable anchor in Prob. 1.49 when $P = 2.5$ kips, knowing that $a = 2$ in. and $b = 6$ in.

PROBLEM 1.49 A steel plate $\frac{1}{4}$ in. thick is embedded in a concrete wall to anchor a high-strength cable as shown. The diameter of the hole in the plate is $\frac{3}{4}$ in., the ultimate strength of the steel used is 36 ksi, and the ultimate bonding stress between plate and concrete is 300 psi. Knowing that a factor of safety of 3.60 is desired when $P = 2.5$ kips, determine (*a*) the required width a of the plate, (*b*) the minimum depth b to which a plate of that width should be embedded in the concrete slab. (Neglect the normal stresses between the concrete and the end of the plate.)

SOLUTION

Based on tension in plate,



$$A = (a - d)t$$

$$= \left(2 - \frac{3}{4}\right)\left(\frac{1}{4}\right) = 0.31250 \text{ in}^2$$

$$P_U = \sigma_U A$$

$$= (36)(0.31250) = 11.2500 \text{ kips}$$

$$F.S. = \frac{P_U}{P} = \frac{11.2500}{3.5} = 4.50$$

Based on shear between plate and concrete slab,

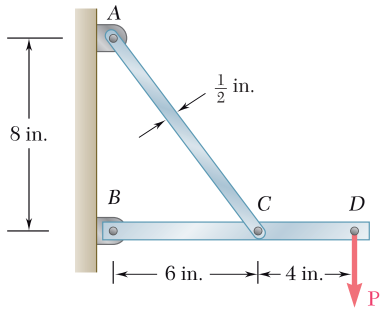
$$A = \text{perimeter} \times \text{depth} = 2(a + t)b = 2\left(2 + \frac{1}{4}\right)(6.0)$$

$$A = 27.0 \text{ in}^2 \qquad \tau_U = 0.300 \text{ ksi}$$

$$P_U = \tau_U A = (0.300)(27.0) = 8.10 \text{ kips}$$

$$F.S. = \frac{P_U}{P} = \frac{8.10}{2.5} = 3.240$$

Actual factor of safety is the smaller value. $F.S. = 3.24$ ◀

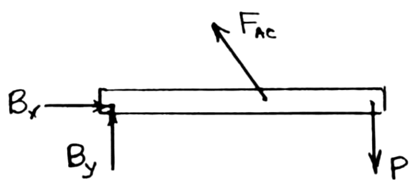


PROBLEM 1.51

Link AC is made of a steel with a 65-ksi ultimate normal stress and has a $\frac{1}{4}\times\frac{1}{2}$-in. uniform rectangular cross section. It is connected to a support at A and to member BCD at C by $\frac{3}{4}$-in.-diameter pins, while member BCD is connected to its support at B by a $\frac{5}{16}$-in.-diameter pin. All of the pins are made of a steel with a 25-ksi ultimate shearing stress and are in single shear. Knowing that a factor of safety of 3.25 is desired, determine the largest load **P** that can be applied at D. Note that link AC is not reinforced around the pin holes.

SOLUTION

Use free body BCD.



$$+\circlearrowleft M_B = 0: \quad (6)\left(\frac{8}{10}F_{AC}\right) - 10P = 0$$

$$P = 0.48F_{AC} \qquad (1)$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x - \frac{6}{10}F_{AC} = 0$$

$$B_x = \frac{6}{10}F_{AC} = 1.25P \longrightarrow$$

$$+\circlearrowleft M_C = 0: \quad -6B_y - 4P = 0$$

$$B_y = -\frac{2}{3}P \qquad \text{i.e.} \quad B_y = \frac{2}{3}P\downarrow$$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{1.25^2 + \left(\frac{2}{3}\right)^2}\,P = 1.41667P \qquad P = 0.70588B \qquad (2)$$

<u>Shear in pins at A and C.</u>

$$F_{AC} = \tau A_{\text{pin}} = \frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = \left(\frac{25}{3.25}\right)\left(\frac{\pi}{4}\right)\left(\frac{3}{8}\right)^2 = 0.84959 \text{ kips}$$

<u>Tension on net section of A and C.</u>

$$F_{AC} = \sigma A_{\text{net}} = \frac{\sigma_U}{F.S.}A_{\text{net}} = \left(\frac{65}{3.25}\right)\left(\frac{1}{4}\right)\left(\frac{1}{2}-\frac{3}{8}\right) = 0.625 \text{ kips}$$

Smaller value of F_{AC} is 0.625 kips.

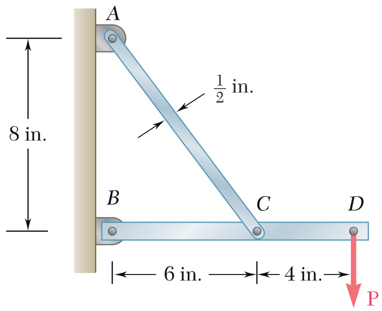
From (1), $\quad P = (0.48)(0.625) = 0.300$ kips

<u>Shear in pin at B.</u> $\quad B = \tau A_{\text{pin}} = \frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = \left(\frac{25}{3.25}\right)\left(\frac{\pi}{4}\right)\left(\frac{5}{16}\right)^2 = 0.58999$ kips

From (2), $\quad P = (0.70588)(0.58999) = 0.416$ kips

Allowable value of P is the smaller value. $\quad P = 0.300$ kips $\quad$ or $\quad P = 300$ lb ◀

PROBLEM 1.52

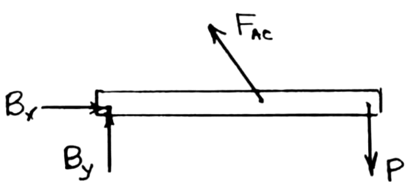


Solve Prob. 1.51, assuming that the structure has been redesigned to use $\frac{5}{16}$-in.-diameter pins at A and C as well as at B and that no other changes have been made.

PROBLEM 1.51 Link AC is made of a steel with a 65-ksi ultimate normal stress and has a $\frac{1}{4} \times \frac{1}{2}$-in. uniform rectangular cross section. It is connected to a support at A and to member BCD at C by $\frac{3}{4}$-in.-diameter pins, while member BCD is connected to its support at B by a $\frac{5}{16}$-in.-diameter pin. All of the pins are made of a steel with a 25-ksi ultimate shearing stress and are in single shear. Knowing that a factor of safety of 3.25 is desired, determine the largest load **P** that can be applied at D. Note that link AC is not reinforced around the pin holes.

SOLUTION

Use free body BCD.



$$+\circlearrowleft M_B = 0: \quad (6)\left(\frac{8}{10}F_{AC}\right) - 10P = 0$$

$$P = 0.48F_{AC} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad B_x - \frac{6}{10}F_{AC} = 0$$

$$B_x = \frac{6}{10}F_{AC} = 1.25P \longrightarrow$$

$$+\circlearrowleft M_C = 0: \quad -6B_y - 4P = 0$$

$$B_y = -\frac{2}{3}P \qquad \text{i.e. } B_y = \frac{2}{3}P\downarrow$$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{1.25^2 + \left(\frac{2}{3}\right)^2}\,P = 1.41667P \qquad P = 0.70583B \qquad (2)$$

Shear in pins at A and C.

$$F_{AC} = \tau A_{\text{pin}} = \frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = \left(\frac{25}{3.25}\right)\left(\frac{\pi}{4}\right)\left(\frac{5}{16}\right)^2 = 0.58999 \text{ kips}$$

Tension on net section of A and C.

$$F_{AC} = \sigma A_{\text{net}} = \frac{\sigma_U}{F.S.}A_{\text{net}} = \left(\frac{65}{3.25}\right)\left(\frac{1}{4}\right)\left(\frac{1}{2} - \frac{5}{16}\right) = 0.9375 \text{ kips}$$

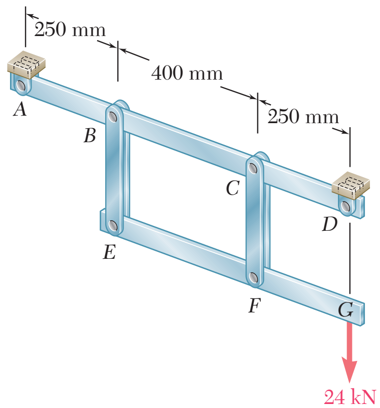
Smaller value of F_{AC} is 0.58999 kips.

From (1), $\quad P = (0.48)(0.58999) = 0.283$ kips

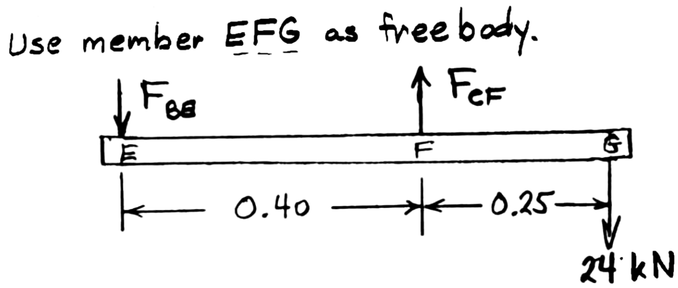
Shear in pin at B. $\quad B = \tau A_{\text{pin}} = \frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = \left(\frac{25}{3.25}\right)\left(\frac{\pi}{4}\right)\left(\frac{5}{16}\right)^2 = 0.58999 \text{ kips}$

From (2), $\quad P = (0.70588)(0.58999) = 0.416$ kips

Allowable value of P is the smaller value. $\quad P = 0.283$ kips $\quad$ or $\quad P = 283$ lb ◀

	**PROBLEM 1.53** Each of the two vertical links *CF* connecting the two horizontal members *AD* and *EG* has a 10 × 40-mm uniform rectangular cross section and is made of a steel with an ultimate strength in tension of 400 MPa, while each of the pins at *C* and *F* has a 20-mm diameter and are made of a steel with an ultimate strength in shear of 150 MPa. Determine the overall factor of safety for the links *CF* and the pins connecting them to the horizontal members.

SOLUTION



$$+\circlearrowleft \Sigma M_E = 0: \quad 0.40F_{CF} - (0.65)(24\times 10^3) = 0$$

$$F_{CF} = 39\times 10^3 \text{ N}$$

Based on tension in links *CF*,

$$A = (b-d)t = (0.040 - 0.02)(0.010) = 200\times 10^{-6}\,\text{m}^2 \quad \text{(one link)}$$

$$F_U = 2\sigma_U A = (2)(400\times 10^6)(200\times 10^{-6}) = 160.0\times 10^3\,\text{N}$$

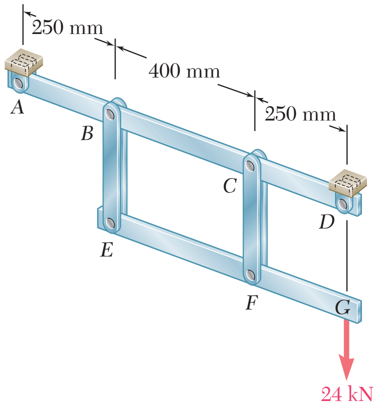
Based on double shear in pins,

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.020)^2 = 314.16\times 10^{-6}\,\text{m}^2$$

$$F_U = 2\tau_U A = (2)(150\times 10^6)(314.16\times 10^{-6}) = 94.248\times 10^3\,\text{N}$$

Actual F_U is smaller value, i.e. $F_U = 94.248\times 10^3$ N

Factor of safety: $F.S. = \dfrac{F_U}{F_{CF}} = \dfrac{94.248\times 10^3}{39\times 10^3}$ $\qquad F.S. = 2.42$ ◀



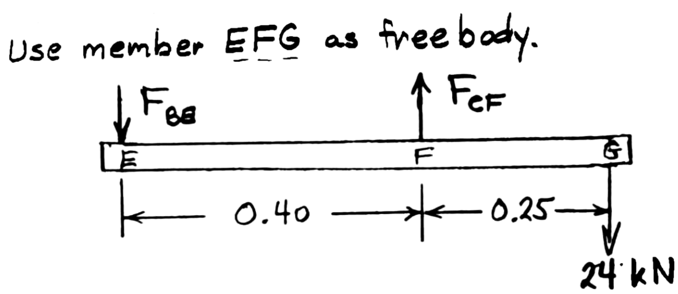
PROBLEM 1.54

Solve Prob. 1.53, assuming that the pins at *C* and *F* have been replaced by pins with a 30-mm diameter.

PROBLEM 1.53 Each of the two vertical links *CF* connecting the two horizontal members *AD* and *EG* has a 10 × 40-mm uniform rectangular cross section and is made of a steel with an ultimate strength in tension of 400 MPa, while each of the pins at *C* and *F* has a 20-mm diameter and are made of a steel with an ultimate strength in shear of 150 MPa. Determine the overall factor of safety for the links *CF* and the pins connecting them to the horizontal members.

SOLUTION

Use member *EFG* as free body.



$$+\circlearrowleft \Sigma M_E = 0: \quad 0.40F_{CF} - (0.65)(24 \times 10^3) = 0$$

$$F_{CF} = 39 \times 10^3 \text{ N}$$

Based on tension in links *CF*,

$$A = (b - d)t = (0.040 - 0.030)(0.010) = 100 \times 10^{-6} \text{ m}^2 \quad \text{(one link)}$$

$$F_U = 2\sigma_U A = (2)(400 \times 10^6)(100 \times 10^{-6}) = 80.0 \times 10^3 \text{ N}$$

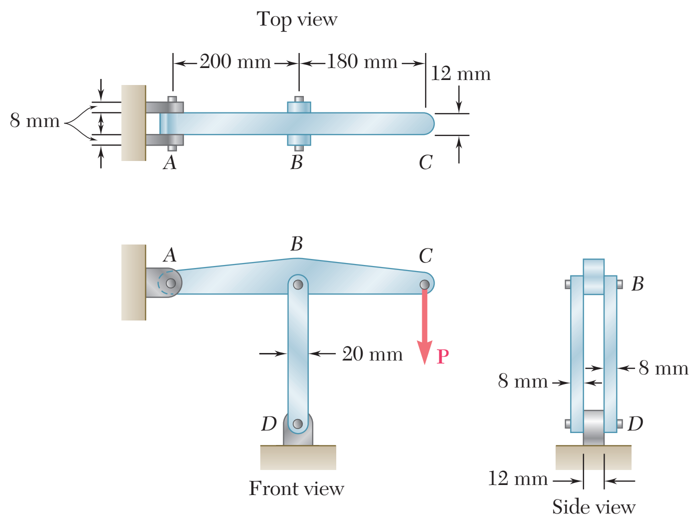
Based on double shear in pins,

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.030)^2 = 706.86 \times 10^{-6} \text{ m}^2$$

$$F_U = 2\tau_U A = (2)(150 \times 10^6)(706.86 \times 10^{-6}) = 212.06 \times 10^3 \text{ N}$$

Actual F_U is smaller value, i.e. $F_U = 80.0 \times 10^3$ N

Factor of safety: $F.S. = \frac{F_U}{F_{CF}} = \frac{80.0 \times 10^3}{39 \times 10^3}$ $\qquad F.S. = 2.05$ ◀

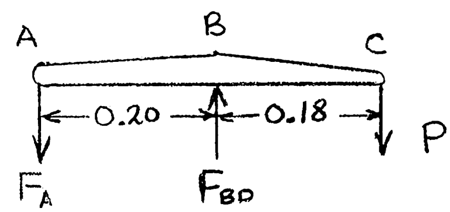


PROBLEM 1.55

In the structure shown, an 8-mm-diameter pin is used at *A*, and 12-mm-diameter pins are used at *B* and *D*. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining *B* and *D*, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.

SOLUTION

Statics: Use *ABC* as free body.



$$+\circlearrowleft \Sigma M_B = 0: \quad 0.20F_A - 0.18P = 0 \qquad P = \frac{10}{9}F_A$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 0.20F_{BD} - 0.38P = 0 \qquad P = \frac{10}{19}F_{BD}$$

Based on double shear in pin *A*, $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.008)^2 = 50.266 \times 10^{-6}\ \text{m}^2$

$$F_A = \frac{2\tau_U A}{F.S.} = \frac{(2)(100 \times 10^6)(50.266 \times 10^{-6})}{3.0} = 3.351 \times 10^3\ \text{N}$$

$$P = \frac{10}{9}F_A = 3.72 \times 10^3\ \text{N}$$

Based on double shear in pins at *B* and *D*, $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.012)^2 = 113.10 \times 10^{-6}\ \text{m}^2$

$$F_{BD} = \frac{2\tau_U A}{F.S.} = \frac{(2)(100 \times 10^6)(113.10 \times 10^{-6})}{3.0} = 7.54 \times 10^3\ \text{N}$$

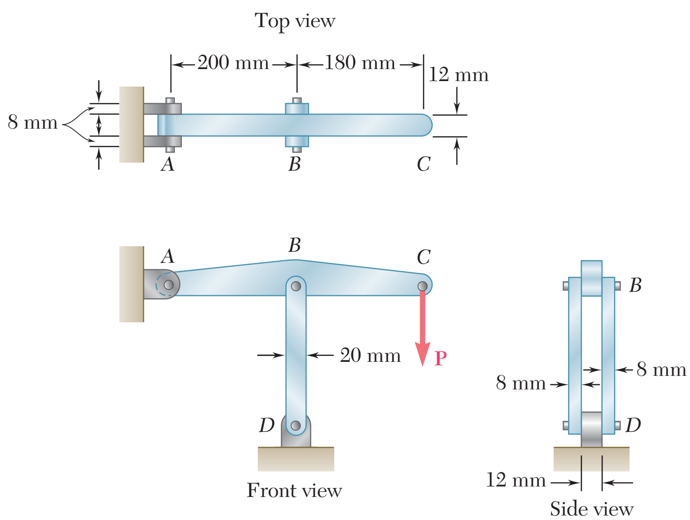
$$P = \frac{10}{19}F_{BD} = 3.97 \times 10^3\ \text{N}$$

Based on compression in links *BD*, for one link, $A = (0.020)(0.008) = 160 \times 10^{-6}\ \text{m}^2$

$$F_{BD} = \frac{2\sigma_U A}{F.S.} = \frac{(2)(250 \times 10^6)(160 \times 10^{-6})}{3.0} = 26.7 \times 10^3\ \text{N}$$

$$P = \frac{10}{19}F_{BD} = 14.04 \times 10^3\ \text{N}$$

Allowable value of *P* is smallest, $\therefore\ P = 3.72 \times 10^3\ \text{N}$ $\qquad P = 3.72\ \text{kN}$ ◀

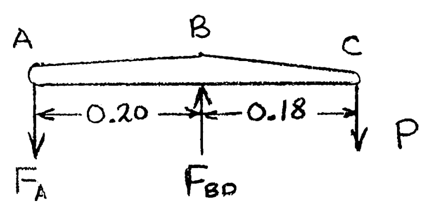


PROBLEM 1.56

In an alternative design for the structure of Prob. 1.55, a pin of 10-mm-diameter is to be used at *A*. Assuming that all other specifications remain unchanged, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.

PROBLEM 1.55 In the structure shown, an 8-mm-diameter pin is used at *A*, and 12-mm-diameter pins are used at *B* and *D*. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining *B* and *D*, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.

SOLUTION



Statics: Use *ABC* as free body.

$$+\circlearrowleft \Sigma M_B = 0: \quad 0.20F_A - 0.18P = 0 \qquad P = \frac{10}{9}F_A$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 0.20F_{BD} - 0.38P = 0 \qquad P = \frac{10}{19}F_{BD}$$

Based on double shear in pin A, $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.010)^2 = 78.54 \times 10^{-6}\ \text{m}^2$

$$F_A = \frac{2\tau_U A}{F.S.} = \frac{(2)(100 \times 10^6)(78.54 \times 10^{-6})}{3.0} = 5.236 \times 10^3\ \text{N}$$

$$P = \frac{10}{9}F_A = 5.82 \times 10^3\ \text{N}$$

Based on double shear in pins at B and D, $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.012)^2 = 113.10 \times 10^{-6}\ \text{m}^2$

$$F_{BD} = \frac{2\tau_U A}{F.S.} = \frac{(2)(100 \times 10^6)(113.10 \times 10^{-6})}{3.0} = 7.54 \times 10^3\ \text{N}$$

$$P = \frac{10}{19}F_{BD} = 3.97 \times 10^3\ \text{N}$$

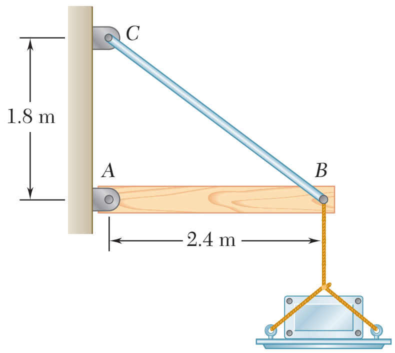
Based on compression in links BD, for one link, $A = (0.020)(0.008) = 160 \times 10^{-6}\ \text{m}^2$

$$F_{BD} = \frac{2\sigma_U A}{F.S.} = \frac{(2)(250 \times 10^6)(160 \times 10^{-6})}{3.0} = 26.7 \times 10^3\ \text{N}$$

$$P = \frac{10}{19}F_{BD} = 14.04 \times 10^3\ \text{N}$$

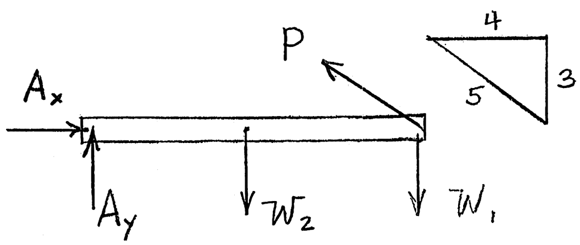
Allowable value of P is smallest, $\therefore\ P = 3.97 \times 10^3\ \text{N}$ $\qquad P = 3.97\ \text{kN}$ ◀

PROBLEM 1.57



A 40-kg platform is attached to the end B of a 50-kg wooden beam AB, which is supported as shown by a pin at A and by a slender steel rod BC with a 12-kN ultimate load. (a) Using the Load and Resistance Factor Design method with a resistance factor $\phi = 0.90$ and load factors $\gamma_D = 1.25$ and $\gamma_L = 1.6$, determine the largest load that can be safely placed on the platform. (b) What is the corresponding conventional factor of safety for rod BC?

SOLUTION



$$+\circlearrowleft \Sigma M_A = 0: \quad (2.4)\frac{3}{5}P - 2.4W_1 - 1.2W_2$$

$$\therefore \quad P = \frac{5}{3}W_1 + \frac{5}{6}W_2$$

For dead loading, $W_1 = (40)(9.81) = 392.4 \text{ N}, \quad W_2 = (50)(9.81) = 490.5 \text{ N}$

$$P_D = \left(\frac{5}{3}\right)(392.4) + \left(\frac{5}{6}\right)(490.5) = 1.0628 \times 10^3 \text{ N}$$

For live loading, $W_1 = mg \quad W_2 = 0 \quad P_L = \frac{5}{3}mg$

From which $m = \frac{3}{5}\frac{P_L}{g}$

Design criterion: $\gamma_D P_D + \gamma_L P_L = \phi P_U$

$$P_L = \frac{\phi P_U - \gamma_D P_D}{\gamma_L} = \frac{(0.90)(12 \times 10^3) - (1.25)(1.0628 \times 10^{-3})}{1.6}$$

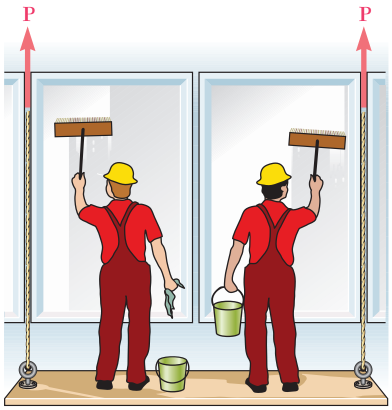
$$= 5.920 \times 10^3 \text{ N}$$

(a) Allowable load. $m = \frac{3}{5}\frac{5.92 \times 10^3}{9.81}$ $\qquad m = 362 \text{ kg}$ ◀

Conventional factor of safety:

$$P = P_D + P_L = 1.0628 \times 10^3 + 5.920 \times 10^3 = 6.983 \times 10^3 \text{ N}$$

(b) $F.S. = \frac{P_U}{P} = \frac{12 \times 10^3}{6.983 \times 10^3}$ $\qquad F.S. = 1.718$ ◀



PROBLEM 1.58

The Load and Resistance Factor Design method is to be used to select the two cables that will raise and lower a platform supporting two window washers. The platform weighs 160 lb and each of the window washers is assumed to weigh 195 lb with equipment. Since these workers are free to move on the platform, 75% of their total weight and the weight of their equipment will be used as the design live load of each cable. (*a*) Assuming a resistance factor $\phi = 0.85$ and load factors $\gamma_D = 1.2$ and $\gamma_L = 1.5$, determine the required minimum ultimate load of one cable. (*b*) What is the corresponding conventional factor of safety for the selected cables?

SOLUTION

$$\gamma_D P_D + \gamma_L P_L = \phi P_U$$

(*a*) $P_U = \dfrac{\gamma_D P_D + \gamma_L P_L}{\phi}$

$$= \frac{(1.2)\left(\frac{1}{2} \times 160\right) + (1.5)\left(\frac{3}{4} \times 2 \times 195\right)}{0.85}$$

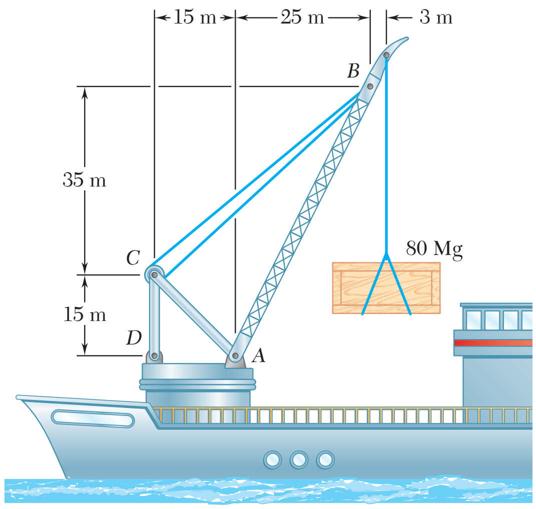
$P_U = 629\ \text{lb}$ ◀

Conventional factor of safety:

$$P = P_D + P_L = \frac{1}{2} \times 160 + 0.75 \times 2 \times 195 = 372.5\ \text{lb}$$

(*b*) $F.S. = \dfrac{P_U}{P} = \dfrac{629}{372.5}$

$F.S. = 1.689$ ◀



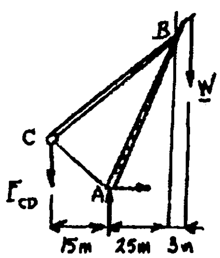
PROBLEM 1.59

In the marine crane shown, link *CD* is known to have a uniform cross section of 50 × 150 mm. For the loading shown, determine`` the normal stress in the central portion of that link.

SOLUTION

Weight of loading: $W = (80 \text{ Mg})(9.81 \text{ m/s}^2) = 784.8 \text{ kN}$

Free Body: Portion *ABC*.

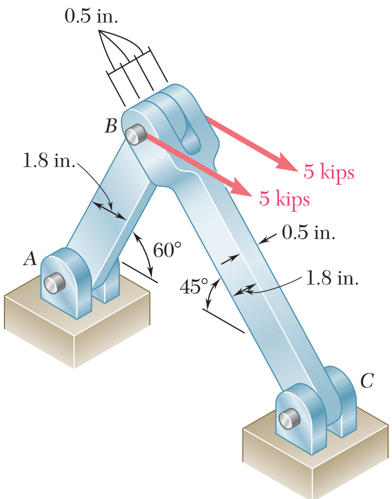


$$+\circlearrowleft \sum M_A = 0: \quad F_{CD}(15 \text{ m}) - W(28 \text{ m}) = 0$$

$$F_{CD} = \frac{28}{15}W = \frac{28}{15}(784.8 \text{ kN})$$

$$F_{CD} = +1465 \text{ kN}$$

$$\sigma_{CD} = \frac{F_{CD}}{A} = \frac{+1465 \times 10^3 \text{ N}}{(0.050 \text{ m})(0.150 \text{ m})} = +195.3 \times 10^6 \text{ Pa} \qquad \sigma_{CD} = +195.3 \text{ MPa} \blacktriangleleft$$

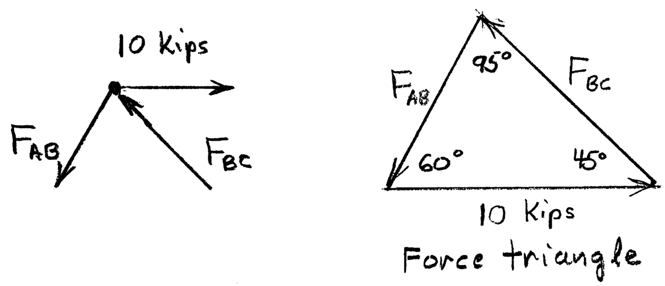


PROBLEM 1.60

Two horizontal 5-kip forces are applied to pin *B* of the assembly shown. Knowing that a pin of 0.8-in. diameter is used at each connection, determine the maximum value of the average normal stress (*a*) in link *AB*, (*b*) in link *BC*.

SOLUTION

Use joint *B* as free body.



Law of Sines:

$$\frac{F_{AB}}{\sin 45^\circ} = \frac{F_{BC}}{\sin 60^\circ} = \frac{10}{\sin 95^\circ}$$

$$F_{AB} = 7.3205 \text{ kips}$$

$$F_{BC} = 8.9658 \text{ kips}$$

Link *AB* is a tension member.

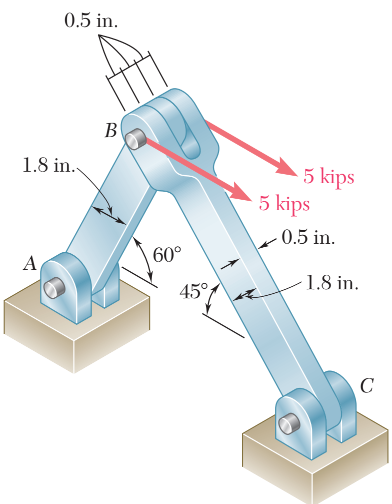
Minimum section at pin: $A_{\text{net}} = (1.8 - 0.8)(0.5) = 0.5 \text{ in}^2$

(*a*) Stress in *AB* : $\sigma_{AB} = \dfrac{F_{AB}}{A_{\text{net}}} = \dfrac{7.3205}{0.5}$ $\sigma_{AB} = 14.64 \text{ ksi}$ ◀

Link *BC* is a compression member.

Cross sectional area is $A = (1.8)(0.5) = 0.9 \text{ in}^2$

(*b*) Stress in *BC*: $\sigma_{BC} = \dfrac{-F_{BC}}{A} = \dfrac{-8.9658}{0.9}$ $\sigma_{BC} = -9.96 \text{ ksi}$ ◀



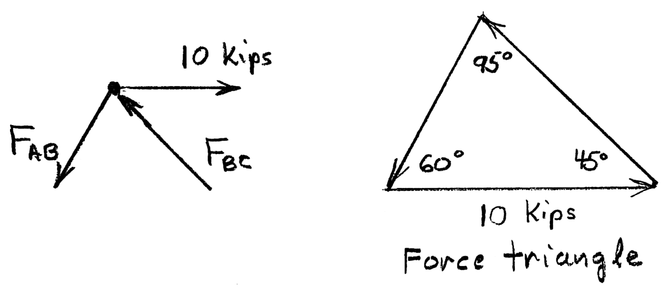
PROBLEM 1.61

For the assembly and loading of Prob. 1.60, determine (*a*) the average shearing stress in the pin at *C*, (*b*) the average bearing stress at *C* in member *BC*, (*c*) the average bearing stress at *B* in member *BC*.

PROBLEM 1.60 Two horizontal 5-kip forces are applied to pin *B* of the assembly shown. Knowing that a pin of 0.8-in. diameter is used at each connection, determine the maximum value of the average normal stress (*a*) in link *AB*, (*b*) in link *BC*.

SOLUTION

Use joint *B* as free body.



Law of Sines:

$$\frac{F_{AB}}{\sin 45°} = \frac{F_{BC}}{\sin 60°} = \frac{10}{\sin 95°} \qquad F_{BC} = 8.9658 \text{ kips}$$

(*a*) Shearing stress in pin at *C*. $\tau = \dfrac{F_{BC}}{2A_P}$

$$A_P = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.8)^2 = 0.5026 \text{ in}^2$$

$$\tau = \frac{8.9658}{(2)(0.5026)} = 8.92 \qquad \tau = 8.92 \text{ ksi} ◀$$

PROBLEM 1.61 *(Continued)*

(*b*) Bearing stress at *C* in member *BC*. $\sigma_b = \frac{F_{BC}}{A}$

$$A = td = (0.5)(0.8) = 0.4\ \text{in}^2$$

$$\sigma_b = \frac{8.9658}{0.4} = 22.4 \qquad \sigma_b = 22.4\ \text{ksi} \ \blacktriangleleft$$

(*c*) Bearing stress at *B* in member *BC*. $\sigma_b = \frac{F_{BC}}{A}$

$$A = 2td = 2(0.5)(0.8) = 0.8\ \text{in}^2$$

$$\sigma_b = \frac{8.9658}{0.8} = 11.21 \qquad \sigma_b = 11.21\ \text{ksi} \ \blacktriangleleft$$

PROBLEM 1.62

Two steel plates are to be held together by means of 16-mm-diameter high-strength steel bolts fitting snugly inside cylindrical brass spacers. Knowing that the average normal stress must not exceed 200 MPa in the bolts and 130 MPa in the spacers, determine the outer diameter of the spacers that yields the most economical and safe design.

SOLUTION

At each bolt location the upper plate is pulled down by the tensile force P_b of the bolt. At the same time, the spacer pushes that plate upward with a compressive force P_s in order to maintain equilibrium.

$$P_b = P_s$$

For the bolt, $\sigma_b = \dfrac{F_b}{A_b} = \dfrac{4P_b}{\pi d_b^2}$ or $P_b = \dfrac{\pi}{4}\sigma_b d_b^2$

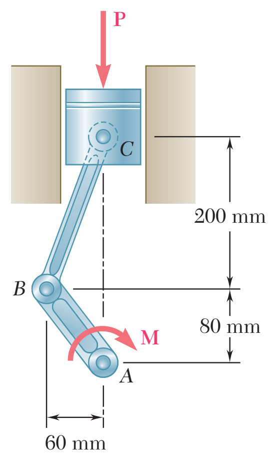
For the spacer, $\sigma_s = \dfrac{P_s}{A_s} = \dfrac{4P_s}{\pi(d_s^2 - d_b^2)}$ or $P_s = \dfrac{\pi}{4}\sigma_s(d_s^2 - d_b^2)$

Equating P_b and P_s,

$$\frac{\pi}{4}\sigma_b d_b^2 = \frac{\pi}{4}\sigma_s(d_s^2 - d_b^2)$$

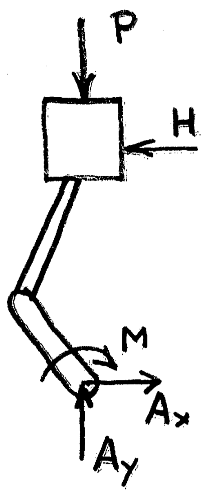
$$d_s = \sqrt{\left(1 + \frac{\sigma_b}{\sigma_s}\right)} d_b = \sqrt{\left(1 + \frac{200}{130}\right)}(16) \qquad d_s = 25.2 \text{ mm} \blacktriangleleft$$

PROBLEM 1.63



A couple **M** of magnitude 1500 N · m is applied to the crank of an engine. For the position shown, determine (*a*) the force **P** required to hold the engine system in equilibrium, (*b*) the average normal stress in the connecting rod *BC*, which has a 450-mm^2 uniform cross section.

SOLUTION

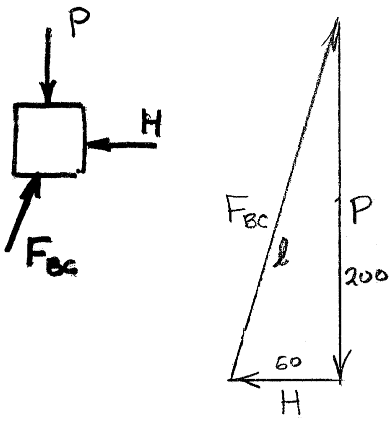


Use piston, rod, and crank together as free body. Add wall reaction H and bearing reactions A_x and A_y.

$$+\circlearrowleft \Sigma M_A = 0: \quad (0.280 \text{ m})H - 1500 \text{ N}\cdot\text{m} = 0$$

$$H = 5.3571 \times 10^3 \text{ N}$$

Use piston alone as free body. Note that rod is a two-force member; hence the direction of force F_{BC} is known. Draw the force triangle and solve for P and F_{BE} by proportions.



$$l = \sqrt{200^2 + 60^2} = 208.81 \text{ mm}$$

$$\frac{P}{H} = \frac{200}{60} \quad \therefore \quad P = 17.86 \times 10^3 \text{ N}$$

(*a*) $P = 17.86$ kN ◀

$$\frac{F_{BC}}{H} = \frac{208.81}{60} \quad \therefore \quad F_{BC} = 18.6436 \times 10^3 \text{ N}$$

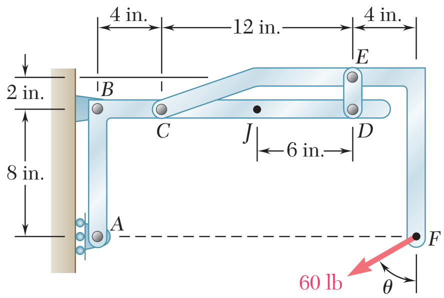
Rod *BC* is a compression member. Its area is

$$450 \text{ mm}^2 = 450 \times 10^{-6} \text{ m}^2$$

Stress:

$$\sigma_{BC} = \frac{-F_{BC}}{A} = \frac{-18.6436 \times 10^3}{450 \times 10^{-6}} = -41.430 \times 10^6 \text{ Pa}$$

(*b*) $\sigma_{BC} = -41.4$ MPa ◀

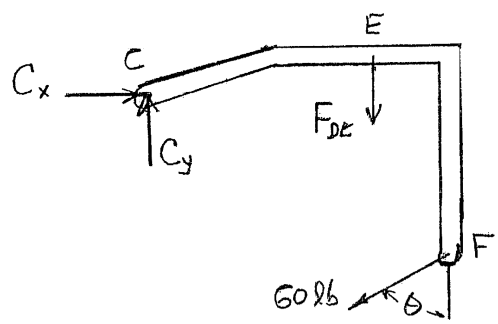


PROBLEM 1.64

Knowing that the link DE is $\frac{1}{8}$ in. thick and 1 in. wide, determine the normal stress in the central portion of that link when (*a*) $\theta = 0°$, (*b*) $\theta = 90°$.

SOLUTION

Use member CEF as a free body.



$$+\circlearrowleft \Sigma M_C = 0: \quad -12\,F_{DE} - (8)(60\sin\theta) - (16)(60\cos\theta) = 0$$

$$F_{DE} = -40\sin\theta - 80\cos\theta \text{ lb}$$

$$A_{DE} = (1)\left(\frac{1}{8}\right) = 0.125 \text{ in}^2$$

$$\sigma_{DE} = \frac{F_{DE}}{A_{DE}}$$

(*a*) $\underline{\theta = 0}$: $F_{DE} = -80$ lb

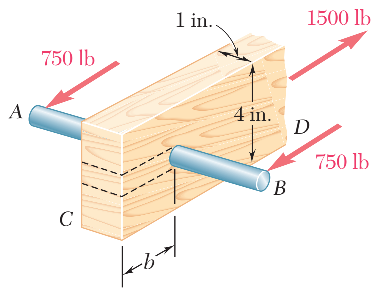
$$\sigma_{DE} = \frac{-80}{0.125}$$

$\sigma_{DE} = -640$ psi ◀

(*b*) $\underline{\theta = 90°}$: $F_{DE} = -40$ lb

$$\sigma_{DE} = \frac{-40}{0.125}$$

$\sigma_{DE} = -320$ psi ◀



PROBLEM 1.65

A $\frac{5}{8}$-in.-diameter steel rod *AB* is fitted to a round hole near end *C* of the wooden member *CD*. For the loading shown, determine (*a*) the maximum average normal stress in the wood, (*b*) the distance *b* for which the average shearing stress is 100 psi on the surfaces indicated by the dashed lines, (*c*) the average bearing stress on the wood.

SOLUTION

(*a*) Maximum normal stress in the wood.

$$A_{\text{net}} = (1)\left(4 - \frac{5}{8}\right) = 3.375 \text{ in}^2$$

$$\sigma = \frac{P}{A_{\text{net}}} = \frac{1500}{3.375} = 444 \text{ psi} \qquad \sigma = 444 \text{ psi} \blacktriangleleft$$

(*b*) Distance b for $\tau = 100$ psi.

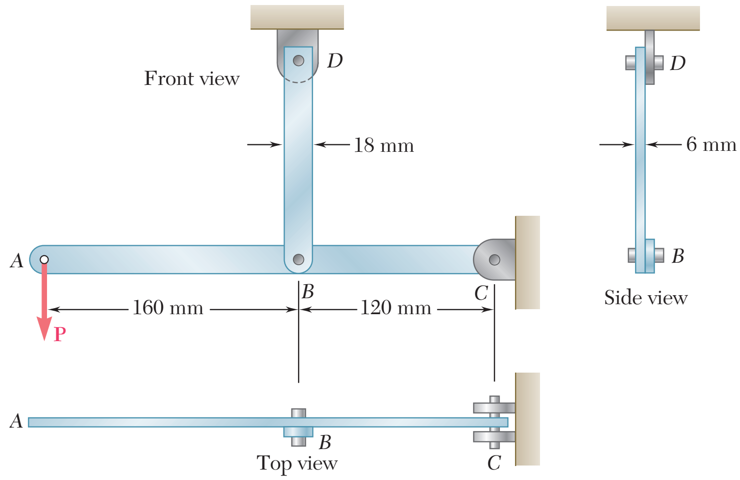
For sheared area see dotted lines.

$$\tau = \frac{P}{A} = \frac{P}{2bt}$$

$$b = \frac{P}{2t\tau} = \frac{1500}{(2)(1)(100)} = 7.50 \text{ in.} \qquad b = 7.50 \text{ in.} \blacktriangleleft$$

(*c*) Average bearing stress on the wood.

$$\sigma_b = \frac{P}{A_b} = \frac{P}{dt} = \frac{1500}{\left(\frac{5}{8}\right)(1)} = 2400 \text{ psi} \qquad \sigma_b = 2400 \text{ psi} \blacktriangleleft$$

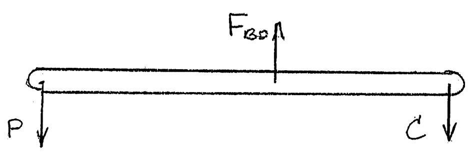


PROBLEM 1.66

In the steel structure shown, a 6-mm-diameter pin is used at C and 10-mm-diameter pins are used at B and D. The ultimate shearing stress is 150 MPa at all connections, and the ultimate normal stress is 400 MPa in link BD. Knowing that a factor of safety of 3.0 is desired, determine the largest load $\mathbf{P}$ that can be applied at A. Note that link BD is not reinforced around the pin holes.

SOLUTION

Use free body ABC.



$$+\circlearrowleft \Sigma M_C = 0: \quad 0.280P - 0.120F_{BD} = 0$$

$$P = \frac{3}{7}F_{BD} \qquad (1)$$

$$+\circlearrowleft \Sigma M_B = 0: \quad 0.160P - 0.120C = 0$$

$$P = \frac{3}{4}C \qquad (2)$$

Tension on net section of link BD:

$$F_{BD} = \sigma A_{\text{net}} = \frac{\sigma_U}{F.S.}A_{\text{net}} = \left(\frac{400 \times 10^6}{3}\right)(6 \times 10^{-3})(18 - 10)(10^{-3}) = 6.40 \times 10^3 \text{ N}$$

Shear in pins at B and D:

$$F_{BD} = \tau A_{\text{pin}} = \frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = \left(\frac{150 \times 10^6}{3}\right)\left(\frac{\pi}{4}\right)(10 \times 10^{-3})^2 = 3.9270 \times 10^3 \text{ N}$$

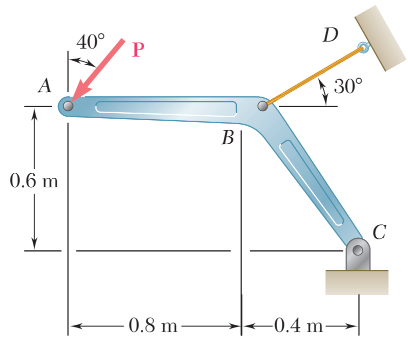
Smaller value of F_{BD} is 3.9270×10^3 N.

From (1), $\quad P = \left(\frac{3}{7}\right)(3.9270 \times 10^3) = 1.683 \times 10^3 \text{ N}$

Shear in pin at C: $\quad C = 2\tau A_{\text{pin}} = 2\frac{\tau_U}{F.S.}\frac{\pi}{4}d^2 = (2)\left(\frac{150 \times 10^6}{3}\right)\left(\frac{\pi}{4}\right)(6 \times 10^{-3})^2 = 2.8274 \times 10^3 \text{ N}$

From (2), $\quad P = \left(\frac{3}{4}\right)(2.8274 \times 10^3) = 2.12 \times 10^3 \text{ N}$

Smaller value of P is allowable value. $\quad P = 1.683 \times 10^3$ N $\qquad P = 1.683$ kN ◀

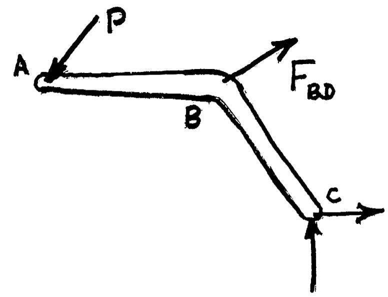


PROBLEM 1.67

Member *ABC*, which is supported by a pin and bracket at *C* and a cable *BD*, was designed to support the 16-kN load **P** as shown. Knowing that the ultimate load for cable *BD* is 100 kN, determine the factor of safety with respect to cable failure.

SOLUTION

Use member *ABC* as a free body, and note that member *BD* is a two-force member.



$$+\circlearrowleft\ \Sigma M_c = 0: \quad (P\cos 40°)(1.2) + (P\sin 40°)(0.6) - (F_{BD}\cos 30°)(0.6) - (F_{BD}\sin 30°)(0.4) = 0$$

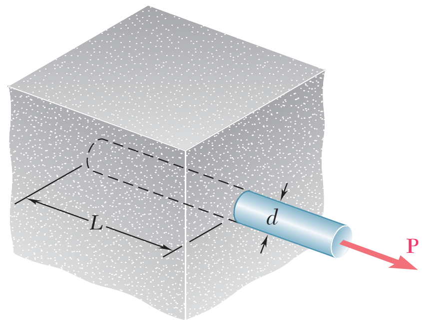
$$1.30493P - 0.71962F_{BD} = 0$$

$$F_{BD} = 1.81335P = (1.81335)(16 \times 10^3) = 29.014 \times 10^3 \text{ N}$$

$$F_U = 100 \times 10^3 \text{ N}$$

$$F.S. = \frac{F_U}{F_{BD}} = \frac{100 \times 10^3}{29.014 \times 10^3}$$

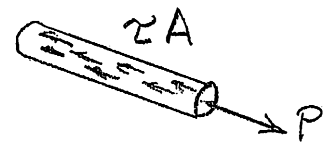
$F.S. = 3.45$ ◀



PROBLEM 1.68

A force **P** is applied as shown to a steel reinforcing bar that has been embedded in a block of concrete. Determine the smallest length L for which the full allowable normal stress in the bar can be developed. Express the result in terms of the diameter d of the bar, the allowable normal stress σ_{all} in the steel, and the average allowable bond stress τ_{all} between the concrete and the cylindrical surface of the bar. (Neglect the normal stresses between the concrete and the end of the bar.)

SOLUTION



For shear, $A = \pi dL$

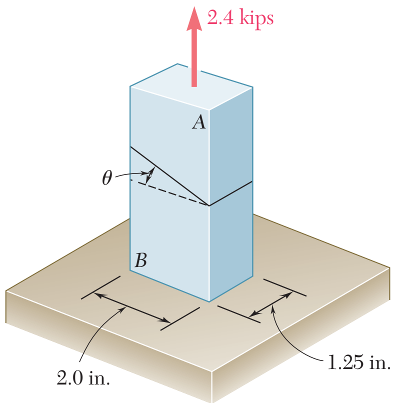
$$P = \tau_{all} A = \tau_{all} \pi dL$$

For tension, $A = \frac{\pi}{4} d^2$

$$P = \sigma_{all} A = \sigma_{all} \left(\frac{\pi}{4} d^2 \right)$$

Equating, $\tau_{all} \pi dL = \sigma_{all} \frac{\pi}{4} d^2$

Solving for L, $L_{min} = \sigma_{all} d / 4\tau_{all}$ ◀



PROBLEM 1.69

The two portions of member AB are glued together along a plane forming an angle θ with the horizontal. Knowing that the ultimate stress for the glued joint is 2.5 ksi in tension and 1.3 ksi in shear, determine (*a*) the value of θ for which the factor of safety of the member is maximum, (*b*) the corresponding value of the factor of safety. (*Hint:* Equate the expressions obtained for the factors of safety with respect to the normal and shearing stresses.)

SOLUTION

$$A_0 = (2.0)(1.25) = 2.50 \text{ in}^2$$

At the optimum angle, $\quad (F.S.)_\sigma = (F.S.)_\tau$

Normal stress: $\quad \sigma = \dfrac{P}{A_0}\cos^2\theta \quad \therefore \quad P_{U,\sigma} = \dfrac{\sigma_U A_0}{\cos^2\theta}$

$$(F.S.)_\sigma = \frac{P_{U,\sigma}}{P} = \frac{\sigma_U A_0}{P\cos^2\theta}$$

Shearing stress: $\tau = \dfrac{P}{A_0}\sin\theta\cos\theta \quad \therefore \quad P_{U,\tau} = \dfrac{\tau_U A_0}{\sin\theta\cos\theta}$

$$(F.S.)_\tau = \frac{P_{U,\tau}}{P} = \frac{\tau_U A_0}{P\sin\theta\cos\theta}$$

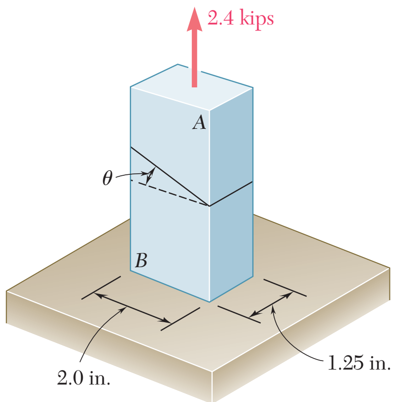
Equating, $\dfrac{\sigma_U A_0}{P\cos^2\theta} = \dfrac{\tau_U A_0}{P\sin\theta\cos\theta}$

Solving, $\quad \dfrac{\sin\theta}{\cos\theta} = \tan\theta = \dfrac{\tau_U}{\sigma_U} = \dfrac{1.3}{2.5} = 0.520 \qquad$ (*a*) $\quad \theta_{\text{opt}} = 27.5° \blacktriangleleft$

(*b*) $\quad P_U = \dfrac{\sigma_U A_0}{\cos^2\theta} = \dfrac{(12.5)(2.50)}{\cos^2 27.5°} = 7.94 \text{ kips}$

$$F.S. = \frac{P_U}{P} = \frac{7.94}{2.4}$$

$F.S. = 3.31 \blacktriangleleft$



PROBLEM 1.70

The two portions of member AB are glued together along a plane forming an angle θ with the horizontal. Knowing that the ultimate stress for the glued joint is 2.5 ksi in tension and 1.3 ksi in shear, determine the range of values of θ for which the factor of safety of the members is at least 3.0.

SOLUTION

$$A_0 = (2.0)(1.25) = 2.50 \text{ in.}^2$$

$$P = 2.4 \text{ kips}$$

$$P_U = (F.S.)P = 7.2 \text{ kips}$$

Based on tensile stress,

$$\sigma_U = \frac{P_U}{A_0}\cos^2\theta$$

$$\cos^2\theta = \frac{\sigma_U A_0}{P_U} = \frac{(2.5)(2.50)}{7.2} = 0.86806$$

$$\cos\theta = 0.93169 \qquad \theta = 21.3° \qquad \theta > 21.3°$$

Based on shearing stress,

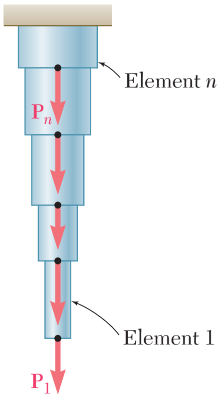
$$\tau_U = \frac{P_U}{A_0}\sin\theta\cos\theta = \frac{P_U}{2A_0}\sin 2\theta$$

$$\sin 2\theta = \frac{2A_0\tau_U}{P_U} = \frac{(2)(2.50)(1.3)}{7.2} = 0.90278$$

$$2\theta = 64.52° \qquad \theta = 32.3° \qquad \theta < 32.3°$$

Hence, $21.3° < \theta < 32.3°$ ◀

PROBLEM 1.C1



A solid steel rod consisting of n cylindrical elements welded together is subjected to the loading shown. The diameter of element i is denoted by d_i and the load applied to its lower end by $\mathbf{P}_i$ with the magnitude P_i of this load being assumed positive if $\mathbf{P}_i$ is directed downward as shown and negative otherwise. (*a*) Write a computer program that can be used with either SI or U.S. customary units to determine the average stress in each element of the rod. (*b*) Use this program to solve Problems 1.1 and 1.3.

SOLUTION

Force in element i:

It is the sum of the forces applied to that element and all lower ones:

$$F_i = \sum_{k=1}^{i} P_k$$

Average stress in element i:

$$\text{Area} = A_i = \frac{1}{4}\pi d_i^2$$

$$\text{Ave. stress} = \frac{F_i}{A_i}$$

Program outputs:

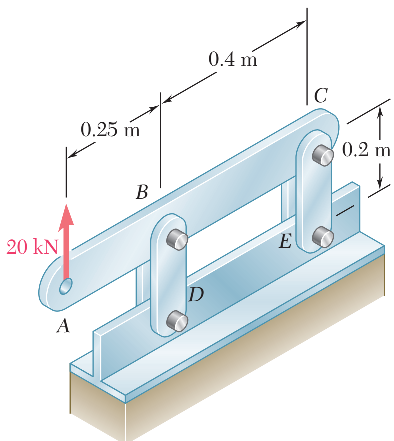
Problem 1.1

Element	**Stress (MPa)**
1	84.883
2	–96.766

Problem 1.3

Element	**Stress (ksi)**
1	22.635
2	17.927

PROBLEM 1.C2

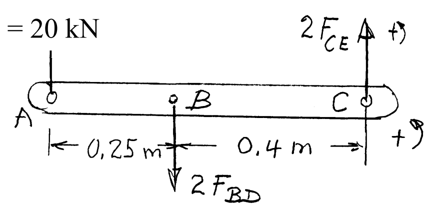


A 20-kN load is applied as shown to the horizontal member *ABC*. Member *ABC* has a 10×50-mm uniform rectangular cross section and is supported by four vertical links, each of 8×36-mm uniform rectangular cross section. Each of the four pins at *A*, *B*, *C*, and *D* has the same diameter *d* and is in double shear. (*a*) Write a computer program to calculate for values of *d* from 10 to 30 mm, using 1-mm increments, (i) the maximum value of the average normal stress in the links connecting pins *B* and *D*, (ii) the average normal stress in the links connecting pins *C* and *E*, (iii) the average shearing stress in pin *B*, (iv) the average shearing stress in pin *C*, (v) the average bearing stress at *B* in member *ABC*, and (vi) the average bearing stress at *C* in member *ABC*. (*b*) Check your program by comparing the values obtained for $d = 16$ mm with the answers given for Probs. 1.7 and 1.27. (*c*) Use this program to find the permissible values of the diameter *d* of the pins, knowing that the allowable values of the normal, shearing, and bearing stresses for the steel used are, respectively, 150 MPa, 90 MPa, and 230 MPa. (*d*) Solve Part *c*, assuming that the thickness of member *ABC* has been reduced from 10 to 8 mm.

SOLUTION

<u>Forces in links.</u>

F.B. diagram of *ABC*:



$$+\circlearrowleft \Sigma M_C = 0: \quad 2F_{BD}(BC) - P(AC) = 0$$

$$F_{BD} = P(AC)/2(BC) \quad \text{(tension)}$$

$$+\circlearrowleft \Sigma M_B = 0: \quad 2F_{CE}(BC) - P(AB) = 0$$

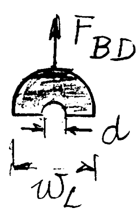
$$F_{CE} = P(AB)/2(BC) \quad \text{(comp.)}$$

(i) <u>Link *BD*.</u>

Thickness $= t_L$

$$A_{BD} = t_L(w_L - d)$$

$$\sigma_{BD} = +F_{BD}/A_{BD}$$



(iii) <u>Pin *B*.</u>

$$\tau_B = F_{BD}/(\pi d^2/4)$$

(v) <u>Bearing stress at *B*.</u>

Thickness of member $AC = t_{AC}$

Sig Bear $B = F_{BD}/(dt_{AC})$

(vi) <u>Bearing stress at *C*.</u>

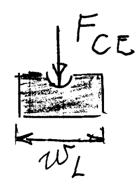
Sig Bear $C = F_{CE}/(dt_{AC})$

(ii) <u>Link *CE*.</u>

Thickness $= t_L$

$$A_{CE} = t_L w_L$$

$$\sigma_{CE} = -F_{CE}/A_{CE}$$



(iv) <u>Pin *C*.</u>

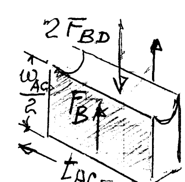
$$\tau_C = F_{CE}/(\pi d^2/4)$$

<u>Shearing stress in *ABC* under Pin *B*.</u>

$$F_B = \tau_{AC} t_{AC}(w_{AC}/2)$$

$$\Sigma F_y = 0: \quad 2F_B = 2F_{BD}$$

$$\tau_{AC} = \frac{2F_{BD}}{\tau_{AC} w_{AC}}$$



Program Outputs

Input data for Parts (*a*), (*b*), (*c*):

$$P = 20 \text{ kN}, \quad AB = 0.25 \text{ m}, \quad BC = 0.40 \text{ m}, \quad AC = 0.65 \text{ m},$$
$$TL = 8 \text{ mm}, \quad WL = 36 \text{ mm}, \quad TAC = 10 \text{ mm}, \quad WAC = 50 \text{ mm}$$

d	Sigma BD	Sigma CE	Tau B	Tau C	SigBear B	SigBear C
10.00	78.13	-21.70	206.90	79.58	325.00	125.00
11.00	81.25	-21.70	170.99	65.77	295.45	113.64
12.00	84.64	-21.70	143.68	55.26	270.83	104.17
13.00	88.32	-21.70	122.43	47.09	250.00	96.15
14.00	92.33	-21.70	105.56	40.60	232.14	89.29
15.00	96.73	-21.70	91.96	35.37	216.67	83.33
16.00	101.56	-21.70	80.82	31.08	203.12	78.13 ← (b)
17.00	106.91	-21.70	71.59	27.54	191.18	73.53
18.00	112.85	-21.70	63.86	24.56	180.56	69.44
19.00	119.49	-21.70	57.31	22.04	171.05	65.79
20.00	126.95	-21.70	51.73	19.89	162.50	62.50
21.00	135.42	-21.70	46.92	18.04	154.76	59.52
22.00	145.09	-21.70	42.75	16.44	147.73	56.82
23.00	156.25	-21.70	39.11	15.04	141.30	54.35
24.00	169.27	-21.70	35.92	13.82	135.42	52.08
25.00	184.66	-21.70	33.10	12.73	130.00	50.00
26.00	203.13	-21.70	30.61	11.77	125.00	48.08
27.00	225.69	-21.70	28.38	10.92	120.37	46.30
28.00	253.91	-21.70	26.39	10.15	116.07	44.64
29.00	290.18	-21.70	24.60	9.46	112.07	43.10
30.00	338.54	-21.70	22.99	8.84	108.33	41.67

(c) Answer: 16 mm ≤ d ≤ 22 mm ◄ (c)

Check: For d = 22 mm, Tau AC = 65 MPa < 90 MPa O.K.

PROBLEM 1.C2 *(Continued)*

Input data for Part (*d*): $P = 20$ kN,

$AB = 0.25$ m, $BC = 0.40$ m,

$AC = 0.65$ m, $TL = 8$ mm, $WL = 36$ mm,

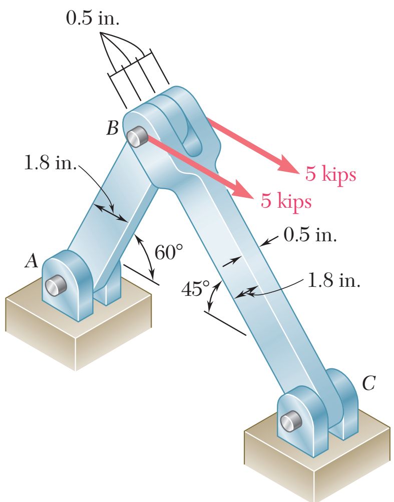
$TAC = 8$ mm, $WAC = 50$ mm

d	Sigma BD	Sigma CE	Tau B	Tau C	SigBear B	SigBear C
10.00	78.13	-21.70	206.90	79.58	406.25	156.25
11.00	81.25	-21.70	170.99	65.77	369.32	142.05
12.00	84.64	-21.70	143.68	55.26	338.54	130.21
13.00	88.32	-21.70	122.43	47.09	312.50	120.19
14.00	92.33	-21.70	105.56	40.60	290.18	111.61
15.00	96.73	-21.70	91.96	35.37	270.83	104.17
16.00	101.56	-21.70	80.82	31.08	253.91	97.66
17.00	106.91	-21.70	71.59	27.54	238.97	91.91
18.00	112.85	-21.70	63.86	24.56	225.69	86.81
19.00	119.49	-21.70	57.31	22.04	213.82	82.24
20.00	126.95	-21.70	51.73	19.89	203.12	78.13
21.00	135.42	-21.70	46.92	18.04	193.45	74.40
22.00	145.09	-21.70	42.75	16.44	184.66	71.02
23.00	156.25	-21.70	39.11	15.04	176.63	67.93
24.00	169.27	-21.70	35.92	13.82	169.27	65.10
25.00	184.66	-21.70	33.10	12.73	162.50	62.50
26.00	203.13	-21.70	30.61	11.77	156.25	60.10
27.00	225.69	-21.70	28.38	10.92	150.46	57.87
28.00	253.91	-21.70	26.39	10.15	145.09	55.80
29.00	290.18	-21.70	24.60	9.46	140.09	53.88
30.00	338.54	-21.70	22.99	8.84	135.42	52.08

(d) Answer: 18 mm ≤ *d* ≤ 22 mm ◀ (d)

Check: For $d = 22$ mm, Tau $AC = 81.25$ MPa < 90 MPa O.K.

PROBLEM 1.C3

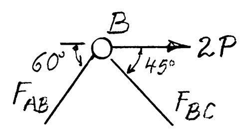


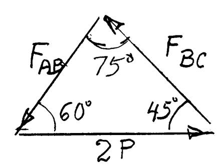
Two horizontal 5-kip forces are applied to Pin *B* of the assembly shown. Each of the three pins at *A*, *B*, and *C* has the same diameter *d* and is double shear. (*a*) Write a computer program to calculate for values of *d* from 0.50 to 1.50 in., using 0.05-in. increments, (i) the maximum value of the average normal stress in member *AB*, (ii) the average normal stress in member *BC*, (iii) the average shearing stress in pin *A*, (iv) the average shearing stress in pin *C*, (v) the average bearing stress at *A* in member *AB*, (vi) the average bearing stress at *C* in member *BC*, and (vii) the average bearing stress at *B* in member *BC*. (*b*) Check your program by comparing the values obtained for $d = 0.8$ in. with the answers given for Problems 1.60 and 1.61. (*c*) Use this program to find the permissible values of the diameter *d* of the pins, knowing that the allowable values of the normal, shearing, and bearing stresses for the steel used are, respectively, 22 ksi, 13 ksi, and 36 ksi. (*d*) Solve Part *c*, assuming that a new design is being investigated in which the thickness and width of the two members are changed, respectively, from 0.5 to 0.3 in. and from 1.8 to 2.4 in.

SOLUTION

<ins>Forces in members *AB* and *BC*.</ins>

Free body: Pin *B*.





From force triangle:

$$\frac{F_{AB}}{\sin 45°} = \frac{F_{BC}}{\sin 60°} = \frac{2P}{\sin 75°}$$

$$F_{AB} = 2P(\sin 45°/\sin 75°)$$

$$F_{BC} = 2P(\sin 60°/\sin 75°)$$

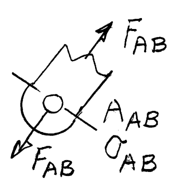
(i) <ins>Max. ave. stress in *AB*.</ins>

Width $= w$

Thickness $= t$

$A_{AB} = (w-d)t$

$\sigma_{AB} = F_{AB}/A_{AB}$



(ii) <ins>Ave. stress in *BC*.</ins>

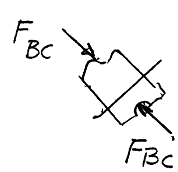
$A_{BC} = wt$

$\sigma_{BC} = F_{BC}/A_{BC}$

(iii) <ins>Pin *A*.</ins>

$\tau_A = (F_{AB}/2)/(\pi d^2/4)$

(iv) <ins>Pin *C*.</ins>



$\tau_C = (F_{BC}/2)/(\pi d^2/4)$

(v) <ins>Bearing stress at *A*.</ins>

Sig Bear $A = F_{AB}/dt$

(vi) <ins>Bearing stress at *C*.</ins>

Sig Bear $C = F_{BC}/dt$

(vii) <ins>Bearing stress at *B* in member *BC*.</ins>

Sig Bear $B = F_{BC}/2dt$

PROBLEM 1.C3 *(Continued)*

Program Outputs

Input data for Parts (*a*), (*b*), (*c*):

$P = 5$ kips, $w = 1.8$ in., $t = 0.5$ in.

D in.	SIGAB ksi	SIGBC ksi	TAUA ksi	TAUC ksi	SIGBRGA ksi	SIGBRGC ksi	SIGBRGB ksi
0.500	11.262	-9.962	18.642	22.831	29.282	35.863	17.932
0.550	11.713	-9.962	15.406	18.869	26.620	32.603	16.301
0.600	12.201	-9.962	12.945	15.855	24.402	29.886	14.943
0.650	12.731	-9.962	11.030	13.510	22.525	27.587	13.793
0.700	13.310	-9.962	9.511	11.649	20.916	25.616	12.808
0.750	13.944	-9.962	8.285	10.147	19.521	23.909	11.954
0.800	14.641	-9.962	7.282	8.918	18.301	22.414	11.207 ← (b)
0.850	15.412	-9.962	6.450	7.900	17.225	21.096	10.548
0.900	16.268	-9.962	5.754	7.047	16.268	19.924	9.962
0.950	17.225	-9.962	5.164	6.324	15.412	18.875	9.438
1.000	18.301	-9.962	4.660	5.708	14.641	17.932	8.966
1.050	19.521	-9.962	4.227	5.177	13.944	17.078	8.539
1.100	20.916	-9.962	3.852	4.717	13.310	16.301	8.151
1.150	22.525	-9.962	3.524	4.316	12.731	15.593	7.796
1.200	24.402	-9.962	3.236	3.964	12.201	14.943	7.471
1.250	26.620	-9.962	2.983	3.653	11.713	14.345	7.173
1.300	29.282	-9.962	2.758	3.377	11.262	13.793	6.897
1.350	32.536	-9.962	2.557	3.132	10.845	13.283	6.641
1.400	36.603	-9.962	2.378	2.912	10.458	12.808	6.404
1.450	41.831	-9.962	2.217	2.715	10.097	12.367	6.183
1.500	48.803	-9.962	2.071	2.537	9.761	11.954	5.977

(c) Answer: 0.70 in. $\leq d \leq$ 1.10 in. ◀ (c)

PROBLEM 1.C3 *(Continued)*

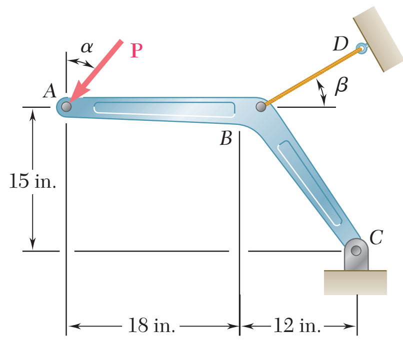
Input data for Part (*d*),

$P = 5$ kips, $w = 2.4$ in., $t = 0.3$ in.

D in.	SIGAB ksi	SIGBC ksi	TAUA ksi	TAUC ksi	SIGBRGA ksi	SIGBRGC ksi	SIGBRGB ksi
0.500	12.843	-12.452	18.642	22.831	48.803	59.772	29.886
0.550	13.190	-12.452	15.406	18.869	44.367	54.338	27.169
0.600	13.556	-12.452	12.945	15.855	40.669	49.810	24.905
0.650	13.944	-12.452	11.030	13.510	37.541	45.978	22.989
0.700	14.354	-12.452	9.511	11.649	34.860	42.694	21.347
0.750	14.789	-12.452	8.285	10.147	32.536	39.848	19.924
0.800	15.251	-12.452	7.282	8.918	30.502	37.357	18.679
0.850	15.743	-12.452	6.450	7.900	28.708	35.160	17.580
0.900	16.268	-12.452	5.754	7.047	27.113	33.206	16.603
0.950	16.829	-12.452	5.164	6.324	25.686	31.459	15.729
1.000	17.430	-12.452	4.660	5.708	24.402	29.886	14.943
1.050	18.075	-12.452	4.227	5.177	23.240	28.463	14.231
1.100	18.771	-12.452	3.852	4.717	22.183	27.169	13.584
1.150	19.521	-12.452	3.524	4.316	21.219	25.988	12.994
1.200	20.335	-12.452	3.236	3.964	20.335	24.905	12.452
1.250	21.219	-12.452	2.983	3.653	19.521	23.909	11.954
1.300	22.183	-12.452	2.758	3.377	18.771	22.989	11.495
1.350	23.240	-12.452	2.557	3.132	18.075	22.138	11.069
1.400	24.402	-12.452	2.378	2.912	17.430	21.347	10.674
1.450	25.686	-12.452	2.217	2.715	16.829	20.611	10.305
1.500	27.113	-12.452	2.071	2.537	16.268	19.924	9.962

(d) Answer: 0.85 in. $\leq d \leq$ 1.25 in. ◀ (d)

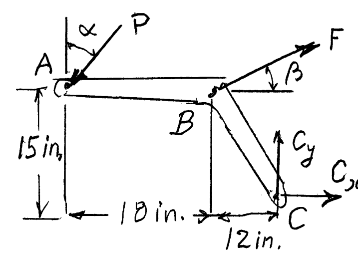
PROBLEM 1.C4



A 4-kip force **P** forming an angle α with the vertical is applied as shown to member *ABC*, which is supported by a pin and bracket at *C* and by a cable *BD* forming an angle β with the horizontal. (*a*) Knowing that the ultimate load of the cable is 25 kips, write a computer program to construct a table of the values of the factor of safety of the cable for values of α and β from 0 to 45°, using increments in α and β corresponding to 0.1 increments in tan α and tan β. (*b*) Check that for any given value of α, the maximum value of the factor of safety is obtained for $\beta = 38.66°$ and explain why. (*c*) Determine the smallest possible value of the factor of safety for $\beta = 38.66°$, as well as the corresponding value of α, and explain the result obtained.

SOLUTION

(a) Draw *F.B.* diagram of *ABC*:



$$+\circlearrowleft \Sigma M_C = 0: \quad (P\sin\alpha)(1.5\text{ in.}) + (P\cos\alpha)(30\text{ in.}) - (F\cos\beta)(15\text{ in.}) - (F\sin\beta)(12\text{ in.}) = 0$$

$$F = P\,\frac{15\sin\alpha + 30\cos\alpha}{15\cos\beta + 12\sin\beta}$$

$$F.S. = F_{\text{ult}}/F$$

Output for $P = 4$ kips and $F_{\text{ult}} = 20$ kips:

VALUES OF FS

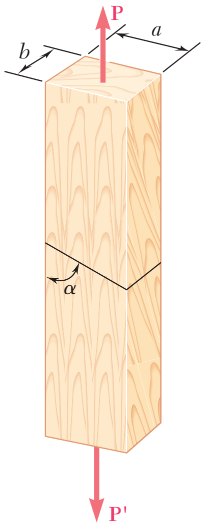
ALPHA \ BETA	0	5.71	11.31	16.70	21.80	26.56	30.96	34.99	38.66	41.99	45.00
0.000	3.125	3.358	3.555	3.712	3.830	3.913	3.966	3.994	4.002	3.995	3.977
5.711	2.991	3.214	3.402	3.552	3.666	3.745	3.796	3.823	3.830	3.824	3.807
11.310	2.897	3.113	3.295	3.441	3.551	3.628	3.677	3.703	3.710	3.704	3.687
16.699	2.837	3.049	3.227	3.370	3.477	3.553	3.600	3.626	3.633	3.627	3.611
21.801	2.805	3.014	3.190	3.331	3.438	3.512	3.560	3.585	3.592	3.586	3.570
26.565	2.795	3.004	3.179	3.320	3.426	3.500	3.547	3.572	3.579	3.573	3.558
30.964	2.803	3.013	3.189	3.330	3.436	3.510	3.558	3.583	3.590	3.584	3.568
34.992	2.826	3.036	3.214	3.356	3.463	3.538	3.586	3.611	3.619	3.612	3.596
38.660	2.859	3.072	3.252	3.395	3.503	3.579	3.628	3.653	3.661	3.655	3.638
41.987	2.899	3.116	3.298	3.444	3.554	3.631	3.680	3.706	3.713	3.707	3.690
45.000	2.946	3.166	3.351	3.499	3.611	3.689	3.739	3.765	3.773	3.767	3.750

↑(b)

(b) When $\beta = 38.66°$, $\tan\beta = 0.8$ and cable *BD* is perpendicular to the lever arm *BC*.

(c) $F.S. = 3.579$ for $\alpha = 26.6°$; *P* is perpendicular to the lever arm *AC*.

Note: The value $F.S. = 3.579$ is the smallest of the values of *F.S.* corresponding to $\beta = 38.66°$ and the largest of those corresponding to $\alpha = 26.6°$. The point $\alpha = 26.6°$, $\beta = 38.66°$ is a "saddle point," or "minimax" of the function $F.S.(\alpha, \beta)$.

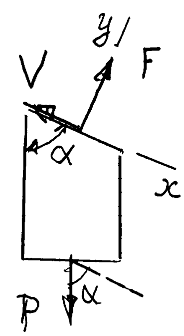


PROBLEM 1.C5

A load **P** is supported as shown by two wooden members of uniform rectangular cross section that are joined by a simple glued scarf splice. (*a*) Denoting by σ_U and τ_U, respectively, the ultimate strength of the joint in tension and in shear, write a computer program which, for given values of *a*, *b*, *P*, σ_U and τ_U, expressed in either SI or U.S. customary units, and for values of α from 5 to 85° at 5° intervals, can be used to calculate (i) the normal stress in the joint, (ii) the shearing stress in the joint, (iii) the factor of safety relative to failure in tension, (iv) the factor of safety relative to failure in shear, and (v) the overall factor of safety for the glued joint. (*b*) Apply this program, using the dimensions and loading of the members of Probs. 1.29 and 1.31, knowing that $\sigma_U = 150$ psi and $\tau_U = 214$ psi for the glue used in Prob. 1.29, and that $\sigma_U = 1.26$ MPa and $\tau_U = 1.50$ MPa for the glue used in Prob. 1.31. (*c*) Verify in each of these two cases that the shearing stress is maximum for $a = 45°$.

SOLUTION

(i) and (ii) Draw the *F.B.* diagram of lower member:



$$\overset{+}{\searrow}\Sigma F_x = 0: \quad -V + P\cos\alpha = 0 \qquad V = P\cos\alpha$$

$$+\nearrow\Sigma F_y = 0: \quad F - P\sin\alpha = 0 \qquad F = P\sin\alpha$$

Area $= ab/\sin\alpha$

Normal stress: $$\sigma = \frac{F}{\text{Area}} = (P/ab)\sin^2\alpha$$

Shearing stress: $$\tau = \frac{V}{\text{Area}} = (P/ab)\sin\alpha\cos\alpha$$

(iii) *F.S.* for tension (normal stresses):

$$FSN = \sigma_U/\sigma$$

(iv) *F.S.* for shear:

$$FSS = \tau_U/\tau$$

(v) Overall *F.S.*:

F.S. = The smaller of *FSN* and *FSS*.

PROBLEM 1.C5 *(Continued)*

Program Outputs

Problem 1.29

$$a = 150 \text{ mm}$$
$$b = 75 \text{ mm}$$
$$P = 11 \text{ kN}$$
$$\sigma_U = 1.26 \text{ MPa}$$
$$\tau_U = 1.50 \text{ MPa}$$

ALPHA	SIG (MPa)	TAU (MPa)	FSN	FSS	FS	
5	0.007	0.085	169.644	17.669	17.669	
10	0.029	0.167	42.736	8.971	8.971	
15	0.065	0.244	19.237	6.136	6.136	
20	0.114	0.314	11.016	4.773	4.773	
25	0.175	0.375	7.215	4.005	4.005	
30	0.244	0.423	5.155	3.543	3.543	
35	0.322	0.459	3.917	3.265	3.265	
40	0.404	0.481	3.119	3.116	3.116	
45	0.489	0.489	2.577	3.068	2.577	◀ (b), (c)
50	0.574	0.481	2.196	3.116	2.196	
55	0.656	0.459	1.920	3.265	1.920	
60	0.733	0.423	1.718	3.543	1.718	
65	0.803	0.375	1.569	4.005	1.569	
70	0.863	0.314	1.459	4.773	1.459	
75	0.912	0.244	1.381	6.136	1.381	
80	0.948	0.167	1.329	8.971	1.329	
85	0.970	0.085	1.298	17.669	1.298	

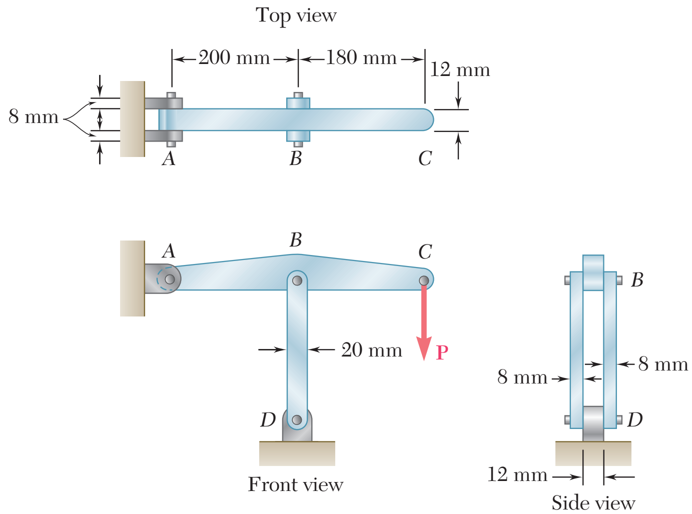
PROBLEM 1.C5 *(Continued)*

Problem 1.31

$$a = 5 \text{ in.}$$
$$b = 3 \text{ in.}$$
$$P = 1400 \text{ lb}$$
$$\sigma_U = 150 \text{ psi}$$
$$\tau_U = 214 \text{ psi}$$

ALPHA	SIG (psi)	TAU (psi)	FSN	FSS	FS	
5	0.709	8.104	211.574	26.408	26.408	
10	2.814	15.961	53.298	13.408	13.408	
15	6.252	23.333	23.992	9.171	9.171	
20	10.918	29.997	13.739	7.134	7.134	
25	16.670	35.749	8.998	5.986	5.986	
30	23.333	40.415	6.429	5.295	5.295	
35	30.706	43.852	4.885	4.880	4.880	
40	38.563	45.958	3.890	4.656	3.890	
45	46.667	46.667	3.214	4.586	3.214	◀ (c)
50	54.770	45.958	2.739	4.656	2.739	
55	62.628	43.852	2.395	4.880	2.395	
60	70.000	40.415	2.143	5.295	2.143	◀ (b)
65	76.663	35.749	1.957	5.986	1.957	
70	82.415	29.997	1.820	7.134	1.820	
75	87.081	23.333	1.723	9.171	1.723	
80	90.519	15.961	1.657	13.408	1.657	
85	92.624	8.104	1.619	26.408	1.619	

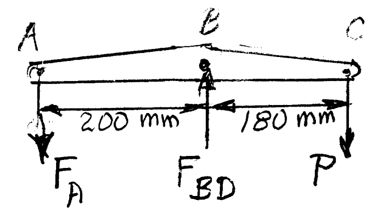
PROBLEM 1.C6



Member *ABC* is supported by a pin and bracket at *A* and by two links, which are pin-connected to the member at *B* and to a fixed support at *D*. (*a*) Write a computer program to calculate the allowable load P_{all} for any given values of (i) the diameter d_1 of the pin at *A*, (ii) the common diameter d_2 of the pins at *B* and *D*, (iii) the ultimate normal stress σ_U in each of the two links, (iv) the ultimate shearing stress τ_U in each of the three pins, and (v) the desired overall factor of safety *F.S.* (*b*) Your program should also indicate which of the following three stresses is critical: the normal stress in the links, the shearing stress in the pin at *A*, or the shearing stress in the pins at *B* and *D*. (*c*) Check your program by using the data of Probs. 1.55 and 1.56, respectively, and comparing the answers obtained for P_{all} with those given in the text. (*d*) Use your program to determine the allowable load P_{all}, as well as which of the stresses is critical, when $d_1 = d_2 = 15$ mm, $\sigma_U = 110$ MPa for aluminum links, $\tau_U = 100$ MPa for steel pins, and $F.S. = 3.2$.

SOLUTION

(a) F.B. diagram of *ABC*:



$$\Sigma M_A = 0: \quad P = \frac{200}{380} F_{BD}$$

$$\Sigma M_B = 0: \quad P = \frac{200}{180} F_A$$

(i) For given d_1 of Pin *A*: $\quad F_A = 2(\tau_U/FS)(\pi d_1^2/4), \quad P_1 = \frac{200}{180} F_A$

(ii) For given d_2 of Pins *B* and *D*: $\quad F_{BD} = 2(\tau_U/FS)(\pi d_2^2/4), \quad P_2 = \frac{200}{380} F_{BD}$

(iii) For ultimate stress in links *BD*: $\quad F_{BD} = 2(\sigma_U/FS)(0.02)(0.008), \quad P_3 = \frac{200}{380} F_{BD}$

(iv) For ultimate shearing stress in pins: P_4 is the smaller of P_1 and P_2.

(v) For desired overall *F.S.*: P_5 is the smaller of P_3 and P_4.

If $P_3 < P_4$, stress is critical in links.

If $P_4 < P_3$ and $P_1 < P_2$, stress is critical in Pin *A*.

If $P_4 < P_3$ and $P_2 < P_1$, stress is critical in Pins *B* and *D*.

PROBLEM 1.C6 *(Continued)*

Program Outputs

(b) Problem 1.55. Data: $d_1 = 8$ mm, $d_2 = 12$ mm, $\sigma_U ,= 250$ MPa, $\tau_U = 100$ MPa, $F.S. = 3.0$

$P_{\text{all}} = 3.72$ kN. Stress in Pin A is critical. ◀

(c) Problem 1.56. Data: $d_1 = 10$ mm, $d_2 = 12$ mm, $\sigma_U = 250$ MPa, $\tau_U = 100$ MPa, $F.S. = 3.0$

$P_{\text{all}} = 3.97$ kN. Stress in Pins B and D is critical. ◀

(d) Data: $d_1 = d_2 = 15$ mm, $\sigma_U = 110$ MPa, $\tau_U = 100$ MPa, $F.S. = 3.2$

$P_{\text{all}} = 5.79$ kN. Stress in links is critical. ◀